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AS PER LATEST CBSE CURRICULUM 2024-25

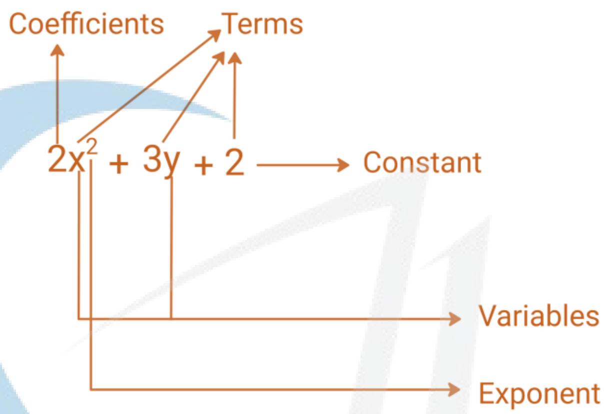
Chapter 2: Polynomial

Concepts Covered:

INTRODUCTION TO POLYNOMIALS

Definition

Polynomial is made up of two terms, namely Poly (meaning "many") and Nominal (meaning "terms."). A polynomial is defined as an expression which is composed of variables, constants and exponents, that are combined using mathematical operations such as addition, subtraction, multiplication and division (No division operation by a variable). Based on the number of terms present in the expression, it is classified as monomial, binomial, and trinomial.



Examples of constants, variables and exponents are as follows:

Constants. **Example:** 1, 2, 3, etc.

Variables. **Example:** g, h, x, y, etc.

Exponents **Example:** 5 in x^5 etc.

Notation

The polynomial function is denoted by P(x) where x represents the variable.

Example: $P(x) = x^2 - 5x + 11$

If the variable is denoted by a, then the function will be P(a)

Degree

The highest power of a variable in a polynomial expression is the degree of the polynomial. The degree of the polynomial with one variable is the highest power of that specific polynomial expression.

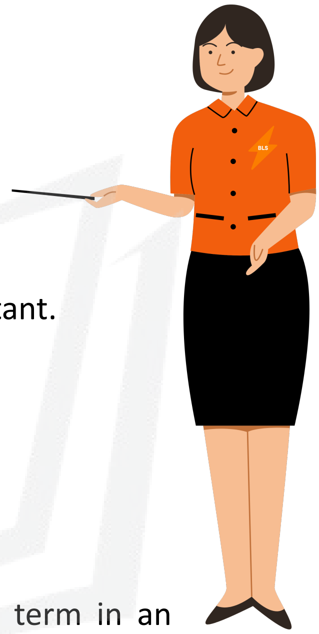
Polynomial Type	Degree	Examples
Constant or Zero Polynomial	0	4
Linear Polynomial	1	$3x + 1$
Quadratic Polynomial	2	$10x^2 + 5x + 1$
Cubic Polynomial	3	$6x^3 + 4x^2 + 3x + 1$
Bi-Quadratic Polynomial	4	$6x^4 + 4x^3 + 2x^2 + 2x + 1$

Terms

The several parts of a polynomial separated by ‘+’ or ‘-‘ operations are called the terms of the expression.

Types of Polynomials Based on there terms

The polynomials are classified into three types based on the total number of terms in that equation. They are as follows:



Zero polynomial:

A zero polynomial is a polynomial that only has one term, which is zero.

Constant polynomial:

A constant polynomial is a polynomial that has only one term, which is a constant.

e.g., $-6, 4, \frac{2}{3}, \frac{-3}{4}$ etc., are still constant polynomial.

Generally, each real number is a constant polynomial.

Monomial:

An expression with only one term is referred to as a monomial. The sole term in an expression needs to be non-zero for it to be a monomial. Some of the examples for monomials are $5x$, 3, $6x^4$, $-3xy$.

Binomial:

A polynomial expression with precisely two terms is referred to as a binomial. Binomial can also be defined as the difference or sum of two or more monomials. Some examples of binomials are:
$5x + 3$, $6x^4 + 17x$, $xy^2 + xy$, etc.

Trinomial:

An expression that has exactly three terms is called a trinomial. Few examples for trinomials are - $8x^3 + 3x^2 + 2x$, $6x^4 + 2x^2 + x$.

Polynomial:

An expression that has more than three terms is called a polynomial. Few examples for polynomials are: $8x^4 + 3x^2 + 2x + 7$, $6x^4 + 2x^2 + x + 8$.

These three types of polynomials can be combined to perform operations related to addition, subtraction, and multiplication. However, not divisible by variable.

Types of Polynomials Based on the degree.

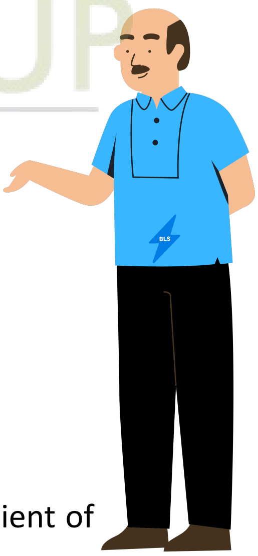
There are various types of polynomials based on the degree of the polynomial. The most common types are:

- Constant Polynomial Function: $P(x) = a = ax^0$
- Zero Polynomial Function: $P(x) = 0$; where all a_i's are zero, $i = 0, 1, 2, 3, \ldots, n$.
- Linear Polynomial Function: $P(x) = ax + b$
- Quadratic Polynomial Function: $P(x) = ax^2 + bx + c$
- Cubic Polynomial Function: $P(x) = ax^3 + bx^2 + cx + d$
- Quartic Polynomial Function: $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

Types of Polynomial			
Constant Polynomial	**Linear Polynomial**	**Quadratic Polynomial**	**Cubic Polynomial**
Polynomial of degree 0	Polynomial of degree 1	Polynomial of degree 2	Polynomial of degree 3
Example: 2, 3, 5… $2 = 2x^0$	**Example:** $x + 2$ $y + 5$ $3u + 4$	**Example:** $2x^2 + 5$ $x^2 + \frac{2}{7}x$ $5x^2 + 2x + \pi$	**Example:** $8x^3$, $2x^3 + x + 1$ $6 - x^3$

Solving Polynomials

To solve polynomials, you typically aim to find the values of the variable that satisfy the polynomial equation. Here are some common methods for solving polynomials,



Linear Polynomial:

Balancing Method:

Consider the linear polynomial $ax + b = 0$, where a and b are coefficients. To solve for x, follow these steps:

Step 1: Move the constant term to the other side of the equation.
Example: Solve the equation $3x - 7 = 0$.
Move -7 to the right side: $3x = 7$.

Step 2: Isolate the variable by dividing both sides of the equation by the coefficient of x.

Example: Divide both sides by 3: $\frac{3x}{3} = \frac{7}{3}$.

Simplifying, you get $x = \frac{7}{3}$.

Step 3: Check the solution by substituting the obtained value back into the original equation.

Example: Substitute $x = \frac{7}{3}$ into 3x - 7 = 0.

You get $3\left(\frac{7}{3}\right) - 7 = 0$, which simplifies to 7 - 7 = 0.

Thus, the solution $x = \frac{7}{3}$ satisfies the equation.

Quadratic Polynomial:

Factorization Method:

If a polynomial can be factored, you can solve it by setting each factor equal to zero and finding the values of the variable.

Example: Solve the equation $x^2 - 5x + 6 = 0$.

Solution: Factoring the polynomial, we have (x - 2) (x - 3) = 0.

Setting each factor equal to zero, we get x - 2 = 0 and x - 3 = 0.

Solving these equations, we find x = 2 and x = 3 as the solutions.

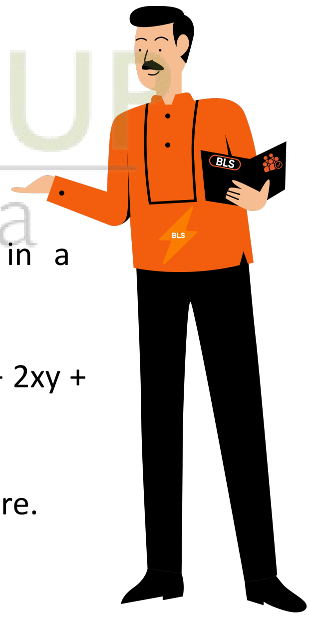
Polynomial Operations

There are four main polynomial operations which are:

- Addition of Polynomials
- Subtraction of Polynomials
- Multiplication of Polynomials
- Division of Polynomials

Addition of Polynomials:

To add polynomials, always add the like terms, i.e., the terms having the same variable and power. The addition of polynomials always results in a polynomial of the same degree.



Example: Find the sum of two polynomials: $5x^3 + 3x^2y + 4xy - 6y^2$, $3x^2 + 7x^2y - 2xy + 4xy^2 - 5$

Solution: First, combine the like terms while leaving the unlike terms as they are.

$(5x^3 + 3x^2y + 4xy - 6y^2) + (3x^2 + 7x^2y - 2xy + 4xy^2 - 5)$

$= 5x^3 + 3x^2 + (3 + 7)x^2y + (4 - 2)xy + 4xy^2 - 6y^2 - 5$

$= 5x^3 + 3x^2 + 10x^2y + 2xy + 4xy^2 – 6y^2 – 5$

Subtraction of Polynomials:

Subtracting polynomials is similar to addition, the only difference being the type of operation. So, subtract the like terms to obtain the solution. It should be noted that subtraction of polynomials also results in a polynomial of the same degree.

Example: Find the difference of two polynomials: $5x^3 + 3x^2y + 4xy – 6y^2$, $3x^2 + 7x^2y – 2xy + 4xy^2 – 5$.

Solution: First, combine the like terms while leaving the unlike terms as they are. Hence,

$(5x^3 + 3x^2y + 4xy – 6y^2) - (3x^2 + 7x^2y – 2xy + 4xy^2 – 5)$

$= 5x^3 - 3x^2 + (3 - 7)x^2y + (4 + 2)xy - 4xy^2 – 6y^2 + 5$

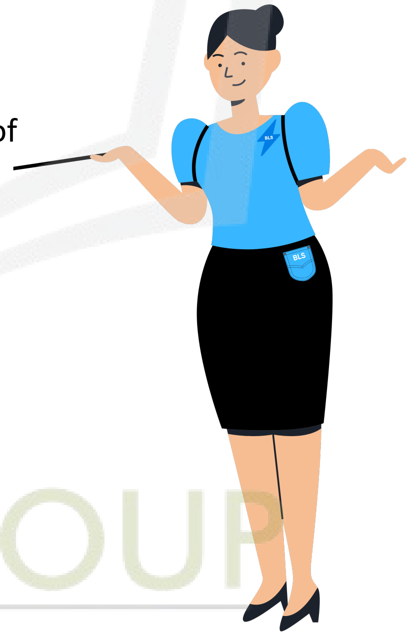
$= 5x^3 -3x^2 - 4x^2y + 6xy - 4xy^2 – 6y^2 + 5$

Multiplication of Polynomials:

Two or more polynomial when multiplied always result in a polynomial of higher degree (unless one of them is a constant polynomial).

Example: Solve: $(6x –3y) \times (2x + 5y)$.

Solution: $\Rightarrow 6x \times (2x + 5y) – 3y \times (2x + 5y)$…Using distributive law of multiplication



$\Rightarrow (12x^2 + 30xy) – (6yx + 15y^2)$…Using distributive law of multiplication

$\Rightarrow 12x^2 + 30xy – 6xy – 15y^2$… as $xy = yx$

Thus, $(6x – 3y) \times (2x + 5y) = 12x^2 + 24xy – 15y^2$

Division of Polynomials:

Division of two polynomial may or may not result in a polynomial.

Polynomial Division Steps:

If a polynomial has more than one term, we use long division method for the same.

Following are the steps for it:

- Write the polynomial in descending order.
- Check the highest power and divide the terms by the same.
- Use the answer in step 2 as the division symbol.
- Now subtract it and bring down the next term.
- Repeat steps 2 to 4 until you have no more terms to carry down.
- Note the final answer, including remainder, will be in the fraction form (last subtract term).

ZEROS OF A POLYNOMIAL

Value of polynomial

The value of polynomial obtained by putting a particular value of the variable is called value of polynomial.

The value of a polynomial p(x) at x = a is denoted by p(a)

Example: $p(x) = 4x^3 - 2x^3 + 2x - 2$

At x = 1,

$p(1) = 4(1)^3 - 2(1)^3 + 2(1) - 2$

$p(1) = 4 - 2 + 2 - 2$

$p(1) = 2 + 2 - 2$

$p(1) = 4 - 2 = 2$

So, 2 is the value of given polynomial p(x) at x = 1

Find the value of each of the following polynomials at the indicated value of variables.

i. $q(y) = 2y^3 - 2y + \sqrt{10}$ at $y = 2$
ii. $p(r) = 4r^2 - 2r + 6$ at $r = a$

i. $q(y) = 2y^3 - 2y + \sqrt{10}$ at $y = 2$

On putting $y = 2$ in $q(y)$ we get,

$q(2) = 2(2)^3 - 2(2) + \sqrt{10}$

$q(2) = 2 \times 8 - 4 + \sqrt{10}$

$q(2) = 16 - 4 + \sqrt{10}$

$q(2) = 12 + \sqrt{10}$, which is the required value of q(y) at 2

ii. $p(r) = 4r^2 - 2r + 6$ at $r = a$

On putting r = a in p(r) we get,

$p(a) = 4(a)^2 - 2(a) + 6$

$p(a) = 4a^2 - 2a + 6$, which is the required value of p(r) at r = a

Zero of a polynomial:

ZEROS OF A POLYNOMIAL

Zero of a polynomial p(x) is a number a, such that p(a) = 0. Zero of a polynomial is also called root of the polynomial, p(x) = 0.

Example: p(x) = x - 3, at x = 3

p(3) = 3 – 3 = 0.

Thus 3 is a zero of polynomial p(x) = x - 2

Zero of a polynomial is special. It is used to find factors of the polynomial.

Example: Find the zero of polynomial 3x + 6

Given polynomial p(x) = 3x + 6

On putting p(x) = 0, we get 3x + 6 = 0

3x = 0 - 6

3x = -6

$$x = \frac{-6}{3} = -2$$

Hence, x = -2 i.e., p(-2) = 0 is the zero of polynomial, 3x + 6.

Check whether -2 and 2 are zeros of the polynomial $t^2 - t - 6$.

The given polynomial is $t^2 - t - 6$...(1)

On putting t = -2 in equation (1) we get,

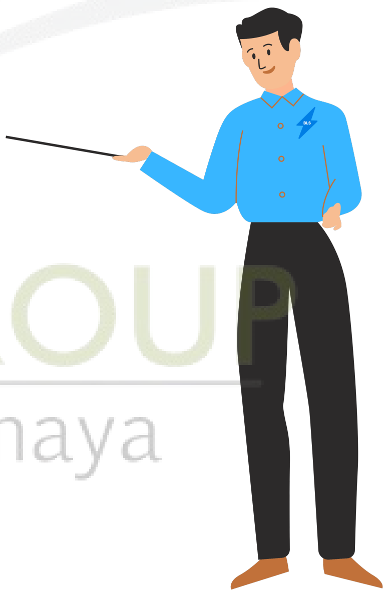
$p(t) = t^2 - t - 6$

$p(-2) = (-2)^2 - (-2) - 6$

p(–2) = 4 + 2 – 6

p(–2) = 6 – 6

p(-2) = 0



Again, on putting t = 2 in equation (1) we get,

$p(t) = t^2 - t - 6$

$p(2) = (2)^2 - (2) - 6$

p(2) = 4 – 2 – 6

p(2) = 2 – 6

p(2) = -4

Therefore, -2 is a zero of the polynomial $t^2 - t - 6$, but 2 is not as the value of p(2) is not equal to zero.

If x = 2 is a root of the polynomial, $f(x) = 2x^2 - 3x + 6a$, Find the value of a.

We know that, $f(x) = 2x^2 - 3x + 6a$... (1)

Given: x = 2 is the root of f(x)

On putting x = 2 in equation (1) we get,

$f(2) = 2(2)^2 - 3(2) + 6a$

$f(2) = 2 \times 4 - 3(2) + 6a$

$f(2) = 8 - 6 + 6a$

$f(2) = 6a + 2$

Now, equate $6a + 2 = 0$

$6a + 2 = 0$

$6a = -2$

$$a = \frac{-2}{6} = \frac{-1}{3}$$

The value of $a = \frac{-1}{3}$

Find the zero (root) of the polynomial in each of the following cases:

$p(x) = x - 7$

$g(x) = 4x + 5$

$h(x) = 3x$

i. $p(x) = x - 7$

The root of $p(x) = x - 7$ is given by $p(x) = 0$

$0 = x - 7$

$x = 7$

Thus $x = 7$ is the root of $p(x) = x - 7$

ii. $g(x) = 4x + 5$

The root of $g(x) = 4x + 5$ is given by $g(x) = 0$

$0 = 4x + 5$

$4x = -5$

$x = -\frac{5}{4}$

Thus $x = -\frac{5}{4}$ is the root of $g(x) = 4x + 5$

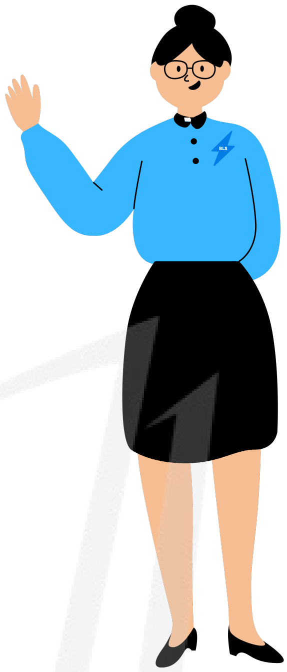
iii. h(x) = 3x

The root of h(x) = 3x is given by h(x) = 0

3x = 0

x = 0

Thus x= 0 is the root of h(x) = 3x.



Important points on zeroes of a polynomial:

1. Zero may be the zero of a polynomial.

Example:

i. $p(x) = x^2$

x = 0

P(0) = 0

ii. p(x) = x + 5

x = 0

p(0) = 0 + 5

p(0) = 5

From the above two examples, we see that zero may or may not be the zero of a polynomial.

2. Every linear polynomial has unique zero.

Example:

i. Consider the linear polynomial p(x) = 3x + 6

For p(x) = 0

3x + 6 = 0

3x = -6

$x = -\frac{6}{3}$

$x = -2$

Therefore for x = -2, the value of a polynomial is 0 and hence, x = -2 is a zero of a given polynomial. There is no other value is possible other than x = -2 for which p(x) = 0 that's how every linear polynomial has unique zero.

3. Zero of a polynomial is called root of the polynomial.

p(x)=0

p(x) = ax + b

therefore ax + b = 0

ax = -b

$$x = -\frac{b}{a}$$

$x = -\frac{b}{a}$ it is the zero of polynomial.

It is also called the root of the polynomial p(x) = ax + b.

4. Every non-zero constant polynomial has no zero.

Example:

Consider the constant polynomial

p(x) = 3 i.e.,

It can be written as $p(x) = 3x^0$...(i)

When we place any number in $3x^0$ still, we get 3

[Because for any value $x^0 = 1$]

Example:

i. Let put $x^0 = 2$ in equation (i) then

$p(x) = 3x^0 = 3 \times 2^0 = 3$

ii. Let put $x^0 = 100$ in equation (i) then

$p(x) = 3x^0 = 3 \times 100^0 = 3$

From the above two example we see that when we place any number in $3x^0$ still, we get 3.

5. Every real number is a zero of the zero polynomial.

Example 1:

$p(x) = ax^2 + bx$

In zero polynomial, all the coefficients are zero hence, a = b = 0

$p(x) = 0.x^2 + 0.x = 0$

For every real number value of x p(x) is zero in this case that's how every real number is a zero of zero polynomial.

Example 2: $p(x) = 0.x^2 + 0.x$

Put x = 1

$p(1) = 0.(1)^2 + 0.(1) = 0$

Example 3: $p(x) = 0.x^2 + 0.x$

Put x = -1

$p(-1) = 0.(-1)^2 + 0.(-1) = 0$

From the above two example we see that every real number is a zero of the zero polynomial

6. A polynomial can have more than one zero

Example: $4x^4 + 0.x^3 – 0.x^2 + 3x + 6$. This polynomial consist of more than one zero.

Consider a polynomial $p(x) = x^2 – 1$.

Let x = 1

For p(x) = 0

$p(x) = x^2 - 1$

$p(1) = (1)^2 - 1$

$p(1) = 1 - 1$

$p(1) = 0$

Let x = - 1

For p(x) = 0

$p(x) = x^2 - 1$

$p(-1) = (-1)^2 - 1$

$p(-1) = 1 - 1$

$p(-1) = 0$

From the above example we see that a polynomial can have more.

REMAINDER THEOREM

Remainder Theorem

Before we start remainder theorem we need to know about factors and multiples, long division algorithm.

Factors and Multiples

If a number is divided by another number exactly, without leaving any non-zero remainder, then the number which divides is called a factor of the number and the number that has been divided is known as the multiple of the number.

Example:

Suppose we divide $3y^3 + y^2 + y$ by y, we get,

$$\frac{3y^3 + y^2 + y}{y} = \frac{3y^3}{y} + \frac{y^2}{y} + \frac{y}{y} = (3y^2 + y + 1)$$

Here, remainder is zero, so y divides $(3y^3 + y^2 + y)$ exactly. Thus, y is a factor of $(3y^3 + y^2 + y)$ and $(3y^3 + y^2 + y)$ is a multiple of y. So, $(3y^2 + y + 1)$ is also a factor of $(3y^3 + y^2 + y)$. Hence, y and $(3y^2 + y + 1)$ are factors of $(3y^3 + y^2 + y) \cdot (3y^3 + y^2 + y)$ is a multiple of y as well as $(3y^2 + y + 1)$.

Long Division Algorithm

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find the polynomial $q(x)$ and $r(x)$such that,

$p(x) = g(x) \times q(x) + r(x)$

$p(x) \rightarrow \text{Dividend}$

$g(x) \rightarrow \text{Divisor}$

$q(x) \rightarrow \text{Quotient}$

$r(x) \rightarrow \text{Remainder}$

Dividend = Divisor × Quotient + Remainder

Here, $p(x)$ when divided by $g(x)$ gives $q(x)$ as quotient and $r(x)$ as remainder.

Where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

This result is known as the Long Division Algorithm

First we will study the method of dividing one polynomial by another polynomial with the help of an example.

Divide $p(x)$ by $g(x)$, where $p(x) = 3x + 4x^2 + 1$ and $g(x) = -1 + x$

i. Firstly, arrange the terms of the dividend and the divisor in descending order of the degrees (also known as writing the polynomial in standard form). We get,

$p(x) = 4x^2 + 3x + 1$ and $g(x) = x - 1$

ii. To obtain the first term of the quotient, we will divide the highest degree term of the dividend, x^2 by the highest degree term of divisor, x. We get the first term of the quotient, 4x and carry out the division process, what remain is (7x + 1).

The first term of the dividend is $4x^2$ and the first term of the divisor is x, since $\frac{4x^2}{x} = 4$ (first term of quotient)

Multiply the divisor (x - 1) by the first term 4x of the quotient and then subtract from dividend.

$$\begin{array}{r|l} & 4x \\ \hline x-1 & 4x^2 + 3x + 1 \\ & 4x^2 - 4x \\ & - \quad\;\; + \\ \hline & \quad\quad\; 7x + 1 \end{array}$$

$[\because 4x(x - 1) = 4x^2 - 4x]$

iii. To obtain the second term of the quotient divide the highest degree term of the new dividend 7x by the highest degree term of the divisor x. We get -7x and what remain is 8. As the degree of 8 is

equal to the degree of the divisor, (x - 1) we will continue the division process.

The first term of the new dividend is 7x and the first term of the divisor is x, since, $\frac{7x}{x} = 7$ second term of quotient.

Multiplying the divisor (x - 1) by the second term of quotient, 7 and then subtracting from the new dividend, we get

$$
\begin{array}{r|l}
 & 4x + 7 \\
\hline
x - 1 & 4x^2 + 3x + 1 \\
 & 4x^2 - 4x \\
 & \quad + \\
\hline
 & \qquad 7x + 1 \\
 & \qquad 7x - 7 \\
 & \qquad - \quad + \\
\hline
 & \qquad\qquad 8
\end{array}
$$

iv. The remainder is 8. As the degree of reminder is less than the divisor, we stop our division process.

Dividend p(x) = $4x^2 + 3x + 1$

Divisor g(x) = x + 1

Quotient q(x) = 4x + 7

Remainder $r(x) = 8$

$p(x) = g(x) \times q(x) + r(x)$

Thus, $4x^2 + 3x + 1 = (x - 1)(4x + 7) + 8$

i.e., Dividend = Divisor × Quotient + Remainder

Find the remainder and quotient when p(t) = $t^3 + t^2 + 2t +$ is divided by t + 2 by actual division method.

$$
\begin{array}{r|l}
 & t^2 - t + 4 \\
\hline
t + 2 & t^3 + t^2 + 2t + 3 \\
 & -(t^3 - 2t^2) \\
\hline
 & \quad -t^2 + 2t + 3 \\
 & \quad -(-t^2 - 2t) \\
 & \qquad + \\
\hline
 & \qquad\quad 4t + 3 \\
 & \qquad\quad -(4t + 8) \\
 & \qquad\qquad - \\
\hline
 & \qquad\qquad -5
\end{array}
$$

Here, the remainder is -5 and quotient $t^2 - t + 4$

Remainder Theorem:

Let p(x) be a polynomial of degree and greater than or equal to 1 $\text{i.e.}, (n \geq 0)$ and a be any real number.

REMAINDER THEOREM

If p(x) is divided by the linear polynomial (x - a), then the remainder is p(a).

Proof:

Let p(x) be any polynomial with degree greater than or equal to 1. We suppose that when p(x) is divided by x – a, the quotient is q(x) and the remainder is r(x),

i.e., p(x) = [(x - a) q(x)] + r(x)...(1)

The degree of (x - a) = 1 and the degree of r(x) is 0.

We see that the degree of r(x) is less than the degree of x – a.

This means that r(x) is a constant, equal to r.

For every value of x, r(x) = r,

On putting r(x) = r in equation (1), we get

P(x) = [(x - a) q(x) + r]...(2)

If x = a, then equation (2) becomes

p(a) = (a – a) q(a) + r

p(a) = r

Find the remainder when the polynomial p(x) = $x^4 - 3x^2 + 2x + 1$ is divided by x^{-2} By remainder theorem, if p(x) is divided by the linear polynomial (x - a), then the remainder is p(a). Therefore, the remainder of the polynomial, p(x) = $x^4 - 3x^2 + 2x + 1$ is p(2).

P(x) = $x^4 - 3x^2 + 2x + 1$

p(2) = $(2)^4 - 3(2)^2 + 2(2) + 1$

p(1) = 16 – 12 + 4 + 1 = 9

The required remainder, p(2) = 9

Find the remainder when the polynomial p(x) = $4x^3 - 12x^2 + 14x - 3$ is divided by $g(x) = x - \frac{1}{2}$

By remainder theorem,

We know that when the polynomial, p(x) is divided by $g(x) = \left(x - \frac{1}{2}\right)$ then the remainder is equal to $p\left(\frac{1}{2}\right)$.

$$p(x) = 4x^3 - 12x^2 + 14x - 3$$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14t\left(\frac{1}{2}\right) - 3$$

$$p\left(\frac{1}{2}\right) = 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 14 \times \frac{1}{2} - 3$$

$$p\left(\frac{1}{2}\right) = \frac{4}{8} - \frac{12}{4} + \frac{14}{2} - 3$$

$$p\left(\frac{1}{2}\right) = \frac{1}{2} - 3 + 7 - 3$$

$$p\left(\frac{1}{2}\right) = \frac{1}{2} + 4 - 3$$

$$p\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{4}{1} - 3$$

$$p\left(\frac{1}{2}\right) = \frac{1+8}{2 \times 1} - 3$$

$$p\left(\frac{1}{2}\right) = \frac{9}{2} - 3 = \frac{3}{2}$$

Hence, required remainder, $p\left(\frac{1}{2}\right) = \frac{3}{2}$.

FACTORIZATION OF POLYNOMIAL

Factor theorem:

Let p(x) be a polynomial of degree $n \geq 1$ and a be any real number such that,

i. p(a) = 0, then (x - a) is a factor of p(x) conversely,
ii. If (x - a) is a factor of p(x), then p(a) = 0.

Proof:

i. Given p(a) = 0

Now, suppose p(x) is divided by (x - a), then quotient is g(x).

By remainder theorem, when p(x) is divided by (x - a), then remainder is p(a).

Dividend = Divisor × Quotient + Remainder

$\therefore p(x) = (x - a) \cdot g(x) + p(a)$

$p(x) = (x - a) \cdot g(x) + 0$[because p(a) = 0]

So, (x - a) is a factor of p(x).

ii. Let (x – a) be a factor of p(x)

On dividing p(x) by (x - a), let g(x) be the quotient.

$\therefore p(x) = (x - a) \cdot g(x)$

On putting x = a, we get

$p(a) = (a - a) \cdot g(a)$

$p(a) = 0 \cdot g(a)$

$p(a) = 0$

Thus, if (x – a) is a factor of p(x), then p(a) = 0

Examine whether x + 1 is a factor of $p(x) = x^3 + 3x^2 + 5x + 6$

Let $p(x) = x^3 + 3x^2 + 5x + 6$

By factor theorem, (x - a) is a factor of p(x) if p(a) = 0.

Therefore, in order to check that (x + 1) is a factor of p(x), it is sufficient to check that it p(-1) = 0 then it is a factor otherwise not.

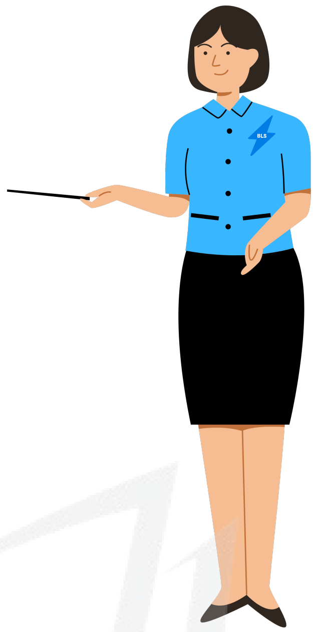
$$p(x) = x^3 + 3x^2 + 5x + 6$$

$p(-1) = (-1)^3 + 3(-1)^2 + 5(-1) + 6$

$p(-1) = -1 + 3 - 5 + 6$

$p(-1) = -6 + 9$

$p(-1) = 3 \neq 0$



Thus, $x + 1$ is NOT a factor of $p(x) = x^3 + 3x^2 + 5x + 6$

Find the value of a if $x – a$ is a factor of $x^3 – a^2x + x + 2$

Hence, $(x - a)$ is a factor of the given polynomial, if $a = -2$.

Let $p(x) = x^3 - a^2 x + x + 2$

By factor theorem, $(x - a)$ is a factor of $p(x)$ if $p(a) = 0$

Now, $p(a) = a^3 - a^2 \times a + a + 2$

$a^3 - a^2 \times a + a + 2 = 0$

$a^3 - a^3 + a + 2 = 0$

$a + 2 = 0$

$a = 0 - 2$

$a = -2$

Factorisation of quadratic polynomial

Quadratic polynomial is represented as $ax^2 + bx + c$, where a, b, c are constants and $a \neq 0$. It can be factorised by different methods.

i. splitting the middle term
ii. By using factor theorem

By splitting middle term

i. Let the factor of quadratic polynomial $ax^2 + b x + c$ be $(px + q)$

and $(rx + s)$. Then

$ax^2 + bx + c = (px + q)(rx + s)$.

$= prx^2 + (ps + qr) x + qs$

Now,

Comparing x^2, x and constant terms, we get

$a = pr$, $b = ps + qr$ and $c = qs$

Here, b is the sum of two numbers ps and q r, whose product is

(ps)(qr) = (pr)(qs) = ac

Thus, to factories a x^2 + b x + c, write b as the sum of two numbers, whose product is ac.

Factorizing $2x^2 + 7x + 3$ by splitting the middle term

Given polynomial is $2x^2 + 7x + 3$.

On comparing with $ax^2 + bx + c$, we get a = 2, b = 7 and c = 3

Now ac = 2 × 3 = 6

So, all possible pairs of factors 6 is 1 and 6, 2 and 3 .

Pair 1 and 6 give 1 + 6 = 7 = b.

$\therefore 2x^2 + 7x + 3 = 2x^2 + (1 + 6)x + 3$

$= 2x^2 + x + 6x + 3$

$= x(2x + 1) + 3(2x + 1)$

$= (2x + 1)(x + 3)$

By using factor theorem

Write the given polynomial $p(x) = ax^2 + bx + c$ in the form

$$p(x) = a\left(x^2 + \frac{b}{a}x + \frac{c}{d}\right) = a \cdot g(x)$$

Where, $g(x) = x^2 + \frac{b}{a}x + \frac{c}{d}$

i.e. firstly make the coefficient of x^2 equal to 1 if it is not 1.

Factories $x^2 - 5x + 6$ by using factor theorem.

Let given polynomial be $f(x) = x^2 - 5x + 6$

Here, the coefficient of x^2 is 1 , so we do not need to write it in the form of a. g(x).

Now, constant term is 6 and all factors of 6 are $\pm 1, \ \pm 2, \ \pm 3$ and ± 6

At x = 2,

$f(2) = (2)^2 - 5(2) + 6$

f(2) = 4 - 10 + 6

f(2) = -6 + 6

f(2) = 0

At x = 3

f(3) = $(3)^2$ - 5(3) + 6

f(3) = 9 - 15 + 6

f(3) = -6 + 6

f(3) = 0

Hence, (x - 2) and (x - 3) are the factors of the given quadratic polynomial.

Factorisation of a cubic polynomial

We use the following steps to factorise a cubic polynomial,

Step 1: Write the given cubic polynomial, p(x) = $ax^3 + bx^2 + cx + d$

As,

$$P(x) = a\left(x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right),$$

Where $g(x) = \left(x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right)$

i.e., first make the coefficient of x^3 equal to 1 if it is not 1, then find the constant term.

Step 2: Find all the possible factors of constant term $\left(\frac{d}{a}\right)$ of $g(x)$.

Step 3: Check at which factor of constant term, p(x) is zero and get one factor of p(x), $(\text{i.e.}, x - \alpha)$

Step 4: Write p(x) as the product of this factor $(x - \alpha)$ and a quadratic polynomial

$\text{i.e.}, p(x) = (x - \alpha)(a_1x^2 + b_1x + c_1)$

Step 5: Apply splitting method of middle term or factor theorem in quadratic polynomial to get the other two factors. Thus, we get all the three factors of given the cubic polynomial.

Using factor theorem, factorise $x^3 + 13x^2 + 32x + 20$

Let $p(x) = x^3 + 13x^2 + 32x + 20$

Here the constant term = 20 and the coefficient of x^3 is 1 .

All possible factors of 20 are $\pm1, \ \pm2, \ \pm4, \ \pm5, \ \pm10$, and ± 20

At x = -1,

$p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33$

$= 0$

So, we find $p(-1) = 0$ So, $(x + 1)$ is a factor of $p(x)$

We can write the given polynomial $x^3 + 13x^2 + 32x + 20$ as, $x^3 +$
$X^2 + 12x^2 + 12x + 20x + 20$

$x^3 + x^2 + 12x^2 + 12x + 20x + 20$

$= x^2(x - 1) + 12x(x + 1) + 20(x + 1)$

$= (x + 1)(x^2 + 12x + 20) \ldots (i)$ [Taking $(x + 1)$common]

Now, $x^2 + 12x + 20$ can be factorised by splitting the middle term, we get,

$x^2 + 12x + 20$

$= x^2 + 10x + 2x + 20$

$= x(x + 10) + 2(x + 10)$

$= (x + 2)(x + 10)$

From equation (i) and (ii) we get,

$x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$

If $p(y) = y^3 - 4y^2 + y + 6$ then show that p(3) = 0 and hence factorise p(y).

Given: $p(y) = y^3 - 4y^2 + y + 6 \ldots\ldots (i)$

Put y = 3 in equation (i), we get

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 3 + 6 = 0$

Since, p(3) = 0, therefore y - 3 is a factor of p(y).

$\therefore p(y) = (y-3)(y^2 - y - 2)$

$\therefore p(y) = (y-3)(y^2 - 2y + y - 2)$

$[\because -2 + 1 = -1 \text{ and } -2 \times 1 = -2]$

$= (y-3)[y(y-2) + 1(y-2])$

$= (y-3)(y+1)(y-2)$

ALGEBRAIC IDENTITIES

An identity is an equality relation A - B where A and B can be variables.

Here, A and B can be differently defined functions but the equality between the two still holds.

Example: $\cos^2x + \sin^2x = 1$ is a trigonometric identity where x is a variable and for any value of x the above result holds true.

So, algebraic identities are algebraic equations that holds true for all values of the variables occurring in it.

Algebraic Identities

$(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a - b)\ (a + b) = a^2 - b^2$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

$(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$

$(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

$(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$

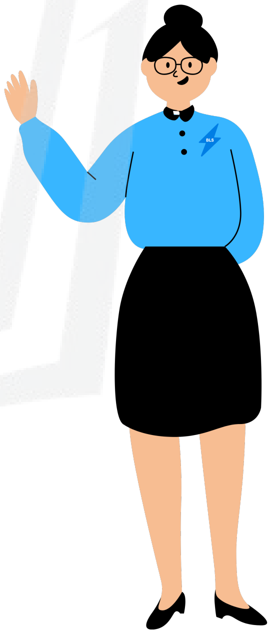
$(a - b)^3 = a^3 - b^3 - 3ab\ (a - b)$

$a^3 + b^3 = (a + b)^3 - 3ab\ (a + b) = (a + b)\ (a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)^3 + 3ab\ (a - b) = (a - b)\ (a^2 + ab + b^2)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)\ (a^2 + b^2 + c^2 - ab - be - ca)$

if, $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$



Application of Algebraic Identities

Example: To find- 99×101 without actual multiplication

Solution: We can write, $99 \times 101 = (100 - 1)\ (100 + 1)$

$= (100)^2 - (1)^2$

$= 10000 - 1 = 9999$

Polynomials

DPP-01

[Topic: Polynomials in One Variable]

Very Short Answer Type Questions

1. Which of the following expressions are polynomials in one variable and which are not? State reason for your answers.
   (i) $x^3 + x$
   (ii) $x + \frac{2}{x} + 3$
   (iii) $\sqrt{3}x + 1$
   (iv) $a^{10} - b^5 + c$
   (v) $2\sqrt{y} + 3y$
2. Write the degree of each of the following polynomials:
   (i) $x^3 - 3x^2 + 1$
   (ii) $\sqrt{2}t - 3$
   (iii) $y^2 + 4y$
   (iv) $2 - y^2 - y^3 + 2y^8$
3. Give an example of trinomial polynomial of degree 27.
4. Classify as linear, quadratic and cubic polynomial,
   (i) s^2
   (ii) $y - y^2 + 1$
   (iii) $1 - x^2$
   (iv) $3 - 2x - x^3$
   (v) $4t + 3$
   (vi) $\sqrt{2}x - x^2 + \frac{1}{\sqrt{3}}x^3$
5. Write the degree of a zero polynomial.
6. Write the degree of the polynomial $p(x) = 4$.
7. Find the degree of the polynomial: $2 - y^2 - y^3 + 2y^7$.
8. Evaluate the degree of the polynomial: $(y^3 - 2)(y^2 + 11)$
9. Determine the degree of the following polynomials:
   (i) $(x - 1)(x - 2x^2 + 3)$
   (ii) $y^3(1 - y^4)$
10. Find the coefficient of x^3 in the polynomial; $p(x) = 6x^4 - \sqrt{3}x^3 - \frac{5}{3}$
11. Find the coefficient of x^2 in $(3x^2 - 5)(4 + 4x^2)$.

Polynomials

DPP-02

[Topic: Zeroes of a Polynomial]

Very Short Answer Type Questions

1. Find the value of the polynomial $5x^2 - 3x + 7$ at:
   (i) $x = 1$
   (ii) $x = -1$
   (iii) 0
   (iv) -2
2. Find the value of the polynomial; $p(z) = 3z^2 - 4z + \sqrt{17}$, when $z = 3$.
3. What is the maximum number of zeroes in a cubic polynomial?
4. What is the value of polynomial $5x - 4x^2 + 3$ at $x = \frac{1}{2}$?
5. If $p(x) = \frac{1}{3}x^3 - \frac{2}{3}x^2 + 5x + 7$, then evaluate $p(3)$.
6. Find zero of the polynomial $p(x) = cx + d$
7. Find the zeroes of polynomial in each of the following:
   (i) $p(x) = x - 5$
   (ii) $g(x) = 2 - 8x$
   (iii) $q(x) = 2x - 7$
   (iv) $h(x) = 2x$
8. Verify whether the following are zeroes of the polynomial, indicate against them.
   (i) $x + 2; x = -2$
   (ii) $x^2 - 2x; x = 0,2$
   (iii) $3x^3 - 2x^2 - x; x = -1$
   (iv) $x^3 - 3\sqrt{3}; x = \sqrt{3}$
   (v) $x^2 - 2x + 1; x = 1$
   (vi) $x^3 - 6x^2 + 11x - 6; x = 1,3$

Short Answer Type Questions-I

9. If -4 is a zero of the polynomial: $p(x) = x^2 + 11x + k$, then find the value of k.
10. Find the value of k, if $x = 2$ is a zero of $p(x) = x^2 + kx + 2k$.
11. If $p(x) = x^3 - x^2 + x + 1$, then find the value of $\frac{p(-1)+p(1)}{2}$
12. If $x = 2$ is a root of the polynomial: $f(x) = 2x^2 - 3x + 7a$, find the value of a.
13. If $p(x) = x^3 - \sqrt{3}x^2 + 2x - 5$. Find $p(3\sqrt{3})$.

Short Answer Type Questions-II

14. If $x = 2$ and $x = 0$ are roots of the polynomial $f(x) = 2x^3 - 5x^2 + ax + b$, find the value of a and b.

15. Find the value of a and b, if $x = 0$ and $x = -1$ are the roots of the polynomial: $f(x) = 2x^3 - 3x^2 + ax + b$.

16. If $p(x) = x^2 - 4x + 3$, evaluate: $p(2) - p(-1) + p\left(\frac{1}{2}\right)$

Polynomials

DPP-03

[Topic: Remainder Theorem]

Very Short Answer Type Questions

1. If a polynomial $f(x)$ is divided by $x-a$, then find the remainder.
2. What is the remainder when x^3-2x^2+x+1 is divided by $(x-1)$?
3. Find the remainder when $p(x)$ is divided by $(ax-b)$.
4. What is the remainder when $x^{31}+31$ is divided by $(x+1)$?
5. What is the remainder when x^2+2x+1 is divided by $(x+1)$?
6. Find the remainder when polynomial $4x^4-3x^3-2x^2+x-7$ is divided by:
   (i) $x-1$
   (ii) $x+1$
   (iii) $2x+1$
   (iv) $1-3x$
   (v) $x-\frac{2}{3}$

Short Answer Type Question-I

7. Find the remainder when: $g(x)=3x^4+2ax^3-5a^2x^2+5x$ is divided by $x-a$.

Short Answer Type Questions-II

8. The polynomial $p(x)=kx^3+9x^2+4x-8$, when divided by $(x+3)$ leaves a remainder $10(1-k)$. Find the value of k.
9. If the polynomial x^3+mx^2+nx+6 has $(x-2)$ as a factor and leaves remainder 3, when divided by $x-3$, find the value of m and n.
10. Divide the polynomial $3x^4-4x^3-3x-1$ by $x-1$ and find its quotient and remainder.
11. Take the polynomial $p(x)=ax^2-bx^3+cx-d$, prove that $b=c$, when it is divided by $x^2=1$.

Long Answer Type Questions

12. Let R_1 and R_2 are the remainders when the polynomial $x^3+2x^2-5ax-7$ and $x^3+ax^2-12x+6$ are divided by $(x+1)$ and $(x-2)$ respectively. If $2R_1+R_2=6$, find the value of a.
13. The polynomials ax^3-3x^2+4 and $2x^3-5x+a$ when divided by $x-2$, leaves the remainders p and q respectively. If $p-2q=4$, find the value of a.

Polynomials

DPP-04

[Topic: Factorisation of Polynomials]

Very Short Answer Type Questions

1. For what value of k, $(x+5)$ is a factor of $p(x) = -x^3 + x + 3k$?
2. If $(x-1)$ is a factor of $p(x) = x^2 + x + k$, then find the value of k.
3. Use the factor theorem to determine whether $q(x)$ is a factor of $p(x)$ in each of the following cases:
   (i) $p(x) = x^3 - 3x^2 + 4x - 4, q(x) = x - 2$
   (ii) $p(x) = 2x^3 + 4x + 6, q(x) = x + 1$
   (iii) $p(x) = 7x^2 - 2\sqrt{8}x - 6, q(x) = x - \sqrt{2}$
   (iv) $p(x) = 3x^3 + x^2 - 20x + 12, q(x) = 3x - 2$
   (v) $p(x) = x^6 - x^5 + x^4 - x^3 + x^2 - x + 1, q(x) = x - 1$

Short Answer Type Questions-I

4. Factorise using factor theorem: $x^2 - 5x + 6$
5. Find the value of m so that $2x - 1$ is a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.
6. Show that $x + 3$ is a factor of $69 + 11x - x^2 + x^3$,
7. If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a.
8. Check whether the polynomial: $p(s) = 3s^3 + s^2 - 20s + 12$ is a multiple of $3s - 2$.
9. Factorise:
   (i) $x^2 + 3\sqrt{2}x + 4$
   [CBSE 2013]
   (ii) $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$
   (iii) $5x^2 + 16x + 3$
   (iv) $\frac{1}{5}x^2 + 2x - 15$

Short Answer Type Questions-II

10. Determine the value of a for which the polynomial $4x^4 - ax^3 + 2x^2 + 4x + 3$ is divisible by $(1 - 2x)$.
11. Factorise $x^3 - 3x^2 - x + 3$ using factor theorem.

Long Answer Type Questions

12. Find the values of a and b so that $(x+1)$ and $(x-2)$ are factors of $x^3 + ax^2 + 2x + b$.
13. If $x - 3$ and $x - \frac{1}{3}$ are factors of the polynomial $px^2 + 3x + r$, show that $p = r$.

14. If $p(x) = x^3 - 4x^2 + x + 6$, then show that $p(3) = 0$ and hence factorise $p(x)$.
15. Factorise using factor theorem: $x^3 + 13x^2 + 32x + 20$
16. Find the values of p and q if $a^2 - 1$ is a factor of $pa^4 - 7a^3 + 9a^2 + qa - 10$.
17. Give a relation between a and b if $(x - 1)$ and $(x + 3)$ are the factors of the polynomial $p(x) = x^3 - ax^2 - 13x + b$. Also find a and b.
18. For the polynomial $a^2(b - c) + b^2(c - a) + c^2(a - b)$, prove that $(a - b)$ is a factor of it, using factor theorem,
19. Using factor theorem prove that $(x - 2)$ is a factor of $2(3 - x) + 3(x - 2) - x$.

Polynomials

DPP-05

[Topic: Algebraic Identities]

Very Short Answer Type Questions

1. Find the value of $(2+\sqrt{3})(2-\sqrt{3})$
2. If $49x^2 - a = \left(7x+\frac{1}{2}\right)\left(7x-\frac{1}{2}\right)$, find the value of a,
3. Factorise:
   (i) $y^3 + 125$
   (ii) $1 - 27x^3$
   (iii) $64a^3 - b^3$
4. If $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$, then find the value of $a+b+c$.
5. If $x+y+z=0$, what is the value of: $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$?
6. Given: $x+\frac{1}{x}=2$, find the value of $x^2+\frac{1}{x^2}$.
7. Given: $x-\frac{1}{x}=5$, find the value of $x^2+\frac{1}{x^2}$.
8. Factorise $(1+3y)^2+(9y^2-1)$.
9. Evaluate the value of $(9)^3+(-3)^3-(6)^3$.

Short Answer Type Questions-I

10. Factorise:
    (i) $8-27a^3-36a+54a^2$
    (ii) $2y^3+y^2-2y-1$
    (iii) $4a^2+b^2+4ab+8a+4b+4$
    (iv) $(x+2)^2+p^2+2p(x+2)$
    (v) $\frac{4}{9}a^2+b^2+\frac{4}{3}ab$
    (vi) $\frac{x^2}{4}-\frac{y^2}{4}$
    (vii) $8x^3y^3+27z^3$
    (viii) $\frac{1}{27}p^3-8q^3$
    (ix) $(x+1)^2-(y-1)^2$
    (x) $(x^2-4x)(x^2-4x-1)-20$
11. Evaluate using identities:
    (i) 95×97
    (ii) 103×96
    (iii) 46×48

(iv) 52×57
(v) $(102)^3$

12. Without actually calculating cubes find the value of:
(i) $(25)^3 + (-17)^3 + (-8)^3$
(ii) $(-12)^3 + 7^3 + 5^3$
(iii) $(0.2)^3 - (0.3)^3 + (0.1)^3$
(iv) $125(x-y)^3 + (5y-3z)^3 + (3z-5x)^3$
(v) $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$

13. Expand:
(i) $\left(3x - \frac{1}{2}y + 2z\right)^2$
(ii) $\left(3\frac{a}{2} + \frac{b}{4} + 2c\right)^2$
(iii) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$
(iv) $\left(4 - \frac{1}{3x}\right)^3$

Short Answer Type Questions-II

14. Give possible expression for the length and breadth of a rectangle whose area is given by:

$$25a^2 - 35a + 12$$

15. Factorise:
(i) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$
(ii) $a^6 - b^6$
(iii) $125x^3 - 27y^3 + 8 + 90xy$
(iv) $x^2 - y^2 - z^2 + 2yz + x + y - z$

16. If $p = 2 - a$, prove that $a^3 + 6ap + p^3 - 8 = 0$

17. Factorise: $(2x-y)^6 - (2x+y)^6$

18. If $x^2 + \frac{1}{x^2} = 38$, then find the value of $x^3 - \frac{1}{x^3}$.

19. Using factor theorem, show that $a+b, b+c$ and $c+a$ are factors of $(a+b+c)^3 - (a^3+b^3+c^3)$.

20. If $a+b = 10$ and $a^2+b^2 = 58$, find the value of a^3+b^3,

21. If $a^2+b^2+c^2 = 90$ and $a+b+c = 20$, then find the value of $ab+bc+ca$.

22. If $a+b+c = 5$ and $ab+bc+ca = 10$, then prove that $a^3+b^3+c^3-3abc = -25$.

23. If $x + \frac{1}{x} = 7$, then find the value of $x^3 + \frac{1}{x^3}$.

24. Verify that $x^3+y^3+z^3-3xyz = \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

25. Factorise: $x^2 + \frac{x}{4} - \frac{1}{8}$

26. Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$

27. What are the possible expression for the dimensions of a cuboid whose volumes are given below?
(i) $2x^2 - 10x$
(ii) $12ky^2 + 8ky - 20k$

28. Factorise:
(i) $2u^3 - 3u^2 - 17u + 30$
(ii) $125a^3 - 27b^3 + 75a^2b - 45ab^2$
(iii) $2x^3 - 13x^2 + 6x + 45$
(iv) $8(x + y)^3 + 27(x - y)^3$
(v) $25x^3y - 121xy^3$
(vi) $x^4 + 4x^2 + 16$
(vii) $(x^2 - 5x + 6)^2 - (x^2 - 6x + 8)^2$
(viii) $2x^3 - xy^2 - y^3$

Long Answer Type Questions

29. Simplify and factorise: $(a + b + c)^2 - (a - b - c)^2 + 4b^2 - 4c^2$.

30. If $a + b + c = 6$ and $ab + bc + ca = 11$, then find the value of $a^3 + b^3 + c^3 - 3abc$.

31. Factorise: $\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$

32. Simplify using identity: $\frac{186\times186\times186+14\times14\times14}{186\times186-186\times14+14\times14}$

33. Factorise: $x^4 - 7x^3 + 9x^2 + 7x - 10$

34. If $x + \frac{1}{x} = \sqrt{7}$, find the value of: (i) $x^2 + \frac{1}{x^2}$
(ii) $x^4 + \frac{1}{x^4}$
(iii) $x^3 + \frac{1}{x^3}$

35. Factorise: $\frac{x^4}{4} + \frac{4}{x^4} + 1$

36. Find the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ if $a + b + c = 3x$.

POLYNOMIALS

INTRODUCTION TO POLYNOMIALS

(Practice Sheet)

1. **What is a polynomial?**
   - **A.** An equation with two terms
   - **B.** An expression with variables, constants, and exponents
   - **C.** A constant value
   - **D.** A linear function

2. **What is the notation for a polynomial function with variable 'a'?**
   - **A.** $P(x) = x^2 - 5x + 11$
   - **B.** $P(a) = a^2 - 5a + 11$
   - **C.** $F(x) = x^2 - 5x + 11$
   - **D.** $P(x) = a^2 - 5a + 11$

3. **What is the degree of the polynomial $4x^3 + 2x^2 - x + 7$?**
   - **A.** 2
   - **B.** 3
   - **C.** 4
   - **D.** 7

4. **Which type of polynomial has exactly three terms?**
   - **A.** Constant polynomial
   - **B.** Linear polynomial
   - **C.** Quadratic polynomial
   - **D.** Trinomial

5. **How do you solve a quadratic polynomial using the factorization method?**
   - **A.** Set each term equal to zero
   - **B.** Divide both sides by the coefficient
   - **C.** Factor the polynomial and set each factor equal to zero
   - **D.** Take the square root of the polynomial

6. Solve $3x – 9$
7. Solve $3x^2 – 6x + x^3 – 18$
8. Determine $y + 1$ is a factor of $y^3 + y^2 + y + 1$
9. Evaluate the polynomial $x^2 + 3$?
10. Evaluate the polynomial $x^2 + 2x + 6$?

(Practice Sheet)

1 **What is the value of the polynomial $q(y) = 2y^3 - 2y + \sqrt{10}$ at $y = 2$?**

A. 12 B. 14

C. $12 + \sqrt{10}$ D. 10

2 **If p(r) = $4r^2$ - 2r + 6, what is the value of p(r) at r = a?**

A. $4a^2 - 2a + 6$ B. 6a + 2

C. 6a D. $a^2 - 2a$

3 **If x = 2 is a root of the polynomial f(x) = $2x^2$ - 3x + 6a, what is the value of a?**

A. -1 B. $-\frac{1}{3}$

C. 1 D. 3

4 **What is the zero of the polynomial p(x) = x - 7?**

A. x = -7 B. x = 0

C. x = 7 D. x = 1

5 **Which of the following is a zero of the polynomial g(x) = 4x + 5?**

A. x = 4 B. x = -5

C. $x = -\frac{5}{4}$ D. x = 0

6 Check whether 0 and 3 are zeroes of the polynomial $x^2 - 3x$.

7 Show that 3 is a zero of the polynomial $x^3 - 8x^2 + 8x + 21$.

8 If $x = -\frac{1}{2}$ is a zero of the polynomial p(x) = $8x^3 - ax^2 - x + 2$, find the value of a?

9 Find Factors and Zeroes of Polynomial f(x) = 5x – 15?

10 Find Factors and Zeroes of Polynomial f(x) = $2x^2 - x - 6$?

POLYNOMIALS

REMAINDER THEOREM

(Practice Sheet)

1 **What is the remainder when $3y^3 + y^2 + y$ is divided by y?**

A. $3y^2 + y + 1$
B. y
C. $3y^3 + y^2$
D. 1

2 **In the division $4x^2 + 3x + 1$by $x - 1$, what is the quotient?**

A. $4x + 7$
B. $4x^2 + 3x + 1$
C. $x - 1$
D. $7x$

3 **If $p(t) = t^3 + t^2 + 2t$ is divided by $t + 2$, what is the remainder?**

A. 4
B. -5
C. $t^2 - t + 4$
D. 2

4 **What is the remainder when $x^4 - 3x^2 + 2x + 1$ is divided by $x - 2$?**

A. 7
B. 9
C. 5
D. 1

5 **What is the degree of the remainder in the division of $3y^3 + y^2 + y$ by y?**

A. 1
B. 2
C. 3
D. 0

6 Divide $2x^2 + 3x + 1$ by $x + 2$ and find quotient and remainder.

7 Find the remainder when $y^3 + y^2 - 2y + 5$ is divided by $y - 5$.

8 When $3x^3 + 2x^2 + 2x + k$ is divided by $x + 2$, the remainder is 4. Calculate the value of k.

9 Write the degree of each of the following polynomials: $3y^3 - x^2y^2 + 4x$

10 Can $(x - 2)$ be the remainder on division of a polynomial $p(x)$ by $(2x + 3)$?

(Practice Sheet)

1 **According to the factor theorem, if p(a) = 0, then:**

A. $(x - a)$ is a factor of p(x)
B. $(x + a)$ is a factor of p(x)
C. $(x \cdot a)$ is a factor of p(x)
D. $(x - a)^2$ is a factor of p(x)

2 **If $(x - 2)$ is a factor of $x^3 - 5x^2 + 3x + 2$, what is p (2)?**

A. 0
B. 1
C. 2
D. 3

3 **To factorize $x^3 - a^2x + x + 2$, the value of a is:**

A. 2
B. -2
C. 1
D. -1

4 **For the polynomial $2x^2 + 7x + 3$, the factorized form using splitting the middle term is:**

A. $(2x + 1)(x + 3)$
B. $(2x + 3)(x + 1)$
C. $(x + 2)(2x + 3)$
D. $(x + 1)(2x + 3)$

5 **If $p(x) = x^3 + 13x^2 + 32x + 20$, which of the following is a factor of p(x)?**

A. $(x - 1)$
B. $(x + 1)$
C. $(x + 2)$
D. $(x + 3)$

6 Divide $P(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$ by $g(x) = x^2 + 3x + 1$ and find the quotient and remainder.

7 Factorize $x^3 + 13x^2 + 32x + 20$

8 Check whether $(x - 1)$ is a factor of the polynomial $p(x) = 6x^3 + 3x^2$

9 Factorise: $x^3 - 2x^2 - x + 2$

10 Find the roots of the equation $2x^2 - 5x + 3 = 0$, by factorisation.

(Practice Sheet)

1 **What is the result of $(a + b)^2$?**

A. $a^2 + 2ab + b^2$ **B.** $a^2 - 2ab + b^2$

C. $a^2 + b^2$ **D.** $a^2 - b^2$

2 **Which algebraic identity is equivalent to $(a - b)^2$?**

A. $a^2 + 2ab + b^2$ **B.** $a^2 - 2ab + b^2$

C. $a^2 + b^2$ **D.** $a^2 - b^2$

3 **What does $(a - b)(a + b)$ equal to?**

A. $a^2 + 2ab + b^2$ **B.** $a^2 - 2ab + b^2$

C. $a^2 + b^2$ **D.** $a^2 - b^2$

4 **What is the expansion of $(a + b + c)^2$?**

A. $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ **B.** $a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$

C. $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$ **D.** $a^2 - b^2 + c^2$

5 **Which expression represents $(a + b)^3$?**

A. $a^3 + b^3 + 3ab$ **B.** $a^3 - b^3 - 3ab$

C. $a^3 + b^3 - 3ab$ **D.** $a^3 - b^3 + 3ab$

6 Expand $(x - 2)^2$.

7 $(a + b)(a^2 - ab + b^2)$.

8 Expand $(2p - q)^3$.

9 How can 99×101 be simplified using algebraic identities?

10 What is the result of $a^3 + b^3 + c^3 - 3abc$?

EXEMPLAR SOLUTIONS

Chapter 2- Polynomials

EXERCISE 2.1

Write the correct answer in each of the following:

1. Which one of the following is a polynomial?

(A) $\frac{x^2}{2} - \frac{2}{x^2}$

(B) $\sqrt{2x} - 1$

(C) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$

(D) $\frac{x-1}{x+1}$

Solution:

(C)

$$x^2 + \frac{3x^{\frac{2}{3}}}{\sqrt{x}} = x^2 + 3x$$

<u>Explanation:</u>

(A)

$$\frac{x^2}{2} - \frac{2}{x^2} = \frac{x^2}{2} - 2x^{-2}$$

The equation contains the term x^2 and $-2x^{-2}$.
Here, the exponent of x in second term = – 2, which is not a whole number.
Hence, the given algebraic expression is not a polynomial.

(B)

$$\sqrt{2x} - 1 = \sqrt{2}x^{\frac{1}{2}} - 1$$

The equation contains the term $\sqrt{2}x^{½}$.
Here, the exponent of x in first term = ½ , which is not a whole number.
Hence, the given algebraic expression is not a polynomial.

(C)

$$x^2 + \frac{3x^{\frac{2}{3}}}{\sqrt{x}} = x^2 + 3x$$

The equation contains the term x^2 and 3x .
Here, the exponent of x in first term and second term= 2and 1 respectively, which is a whole number.
Hence, the given algebraic expression is a polynomial.

(D)

$$\frac{x-1}{x+1}$$

The equation is a rational function.
Here, the given equation is not in the standard form of polynomial.
Hence, the given algebraic expression is not a polynomial.

Hence, option C is the correct answer

2. $\sqrt{2}$ is a polynomial of degree

(A) 2
(B) 0
(C) 1
(D) ½

Solution:

(B) 0

Explanation:

$\sqrt{2}$ can be written as $\sqrt{2}x^0$

i.e., $\sqrt{2} = \sqrt{2}x^0$

Therefore, the degree of the polynomial = 0

Hence, option B is the correct answer

3. Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is

(A) 4
(B) 5
(C) 3
(D) 7

Solution:

(A) 4

Explanation:

Degree of a polynomial = Highest power of the variable in a polynomial.

The highest power of variable x in the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is 4.

Therefore, degree of the polynomial of $4x^4 + 0x^3 + 0x^5 + 5x + 7 = 4$

Hence, option A is the correct answer

4. Degree of the zero polynomial is

(A) 0
(B) 1
(C) Any natural number
(D) Not defined

Solution:

(D) Not defined

Explanation:

Degree of a zero polynomial is not defined.

Hence, option D is the correct answer

5. If $p(x)= x^2 – 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to

(A) 0
(B) 1
(C) $4\sqrt{2}$
(D) $8\sqrt{2} +1$

Solution:

(B) 1

Explanation:

According to the question,

$p(x) = x^2 - 2\sqrt{2}x + 1$

To get $p(2\sqrt{2})$,

We substitute $x = 2\sqrt{2}$,

$p(2\sqrt{2}) = (2\sqrt{2})^2 - (2\sqrt{2} \times (2\sqrt{2})) + 1$

$= (4 \times 2) - (4 \times 2) + 1$

$= 8 - 8 + 1$

$= 1$

Hence, option B is the correct answer

6. The value of the polynomial $5x - 4x^2 + 3$, when $x = -1$ is

(A) – 6

(B) 6

(C) 2

(D) – 2

Solution:

(A) – 6

Explanation:

According to the question,

$p(x) = 5x - 4x^2 + 3$

To get $p(-1)$,

We substitute $x = -1$,

$p(-1) = 5(-1) - 4(-1)^2 + 3$

$= 5(-1) - 4(1) + 3$

$= -5 - 4 + 3$

$= -9 + 3$

$= -6$

Hence, option A is the correct answer

7. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

(A) 3

(B) 2x

(C) 0

(D) 6

Solution:

(D) 6

Explanation:

$p(x) = x + 3$

$p(-x) = -x + 3$

Therefore,

$p(x) + p(-x) = (x + 3) + (-x + 3)$

$= x + 3 - x + 3$

$= 6$

Hence, option D is the correct answer

8. Zero of the zero polynomial is

(A) 0

(B) 1

(C) Any real number

(D) Not defined

Solution:

(C) Any real number

Explanation:

Zero polynomial is a constant polynomial whose coefficients are all equal to 0.

Zero of a polynomial is the value of the variable that makes the polynomial equal to zero.

Therefore, zero of the zero polynomial is any real number.

Hence, option C is the correct answer

9. Zero of the polynomial $p(x) = 2x + 5$ is

(A) – 2/5

(B) – 5/2

(C) 2/5

(D) 5/2

Solution:

(B) – 5/2

Explanation:

Zero of the polynomial $\Rightarrow p(x) = 0$

$p(x) = 0$

$2x + 5 = 0$

$2x = -5$

$x = -5/2$

Hence, option B is the correct answer

10. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is

(A) 2

(B) ½

(C) – ½

(D) –2

Solution:

(B) ½

Explanation:

Zero of the polynomial $\Rightarrow p(x) = 0$

$p(x) = 0$

$2x^2 + 7x - 4 = 0$

$2x^2 - 1x + 8x - 4 = 0$

$x(2x - 1) + 4(2x - 1) = 0$

$(x + 4)(2x - 1) = 0$

Consider, $x + 4$

$x + 4 = 0$

$x = -4$

Consider, $2x - 1$

$2x - 1 = 0$

$2x = 1$

$x = ½$

Hence, option B is the correct answer

EXERCISE 2.2

1. Which of the following expressions are polynomials? Justify your answer:

(i) 8 (ii) $\sqrt{3}x^2 - 2x$ (iii) $1-\sqrt{5x}$

(iv) $\dfrac{1}{5x^{-2}} + 5x + 7$ (v) $\dfrac{(x-2)(x-4)}{x}$ (vi) $\dfrac{1}{x+1}$

(vii) $\dfrac{1}{7}a^3 - \dfrac{2}{\sqrt{3}}a^2 + 4a - 7$ (viii) $\dfrac{1}{2x}$

Solution:

(i) 8

8 can be written as $8x^0$.

i.e., $8 = 8x^0$,

Here, the power of $x = 0$, which is a whole number.

Hence, 8 is a polynomial.

(ii) $\sqrt{3}x^2 - 2x$

$\sqrt{3}x^2 - 2x$

Here, the power of x are 2 and 1 respectively

2 and 1 both are whole numbers.

Hence, $\sqrt{3}x^2 - 2x$ is a polynomial.

(iii) $1 - \sqrt{(5x)}$

$1 - \sqrt{5}\sqrt{x} = 1 - \sqrt{5}\, x^{½}$

Here, the power of x = ½, which is not a whole number.

Hence, $1 - \sqrt{5x}$ is not a polynomial

(iv)

$$\frac{1}{5x^{-2}} + 5x + 7$$

$1/5x^{-2} + 5x + 7 = 5x^2 + 5x + 7$

Here, the power of x are 2 and 1 respectively

2 and 1 both are whole numbers.

Hence, $1/5x^{-2} + 5x + 7$ is a polynomial.

(v)

$$\frac{(x-2)(x-4)}{x}$$

$((x-2)(x-4))/x$ $= (x^2 - 4x - 2x + 8)/x$

$= (x^2 - 6x + 8)/x$

$= x - 6 + (8/x)$

$= x - 6 + 8x^{-1}$

Here, the power of x = – 1, which is not a whole number, but a negative number.
Hence, ((x – 2)(x – 4))/x is not a polynomial

(vi)

$$\frac{1}{x+1}$$

$1/(x+1) = (x+1)^{-1}$
Here, the power of x is not a whole number.
Hence, 1/(x+1) is not a polynomial

(vii)

$$\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$$

$(1/7)a^3 - (2/\sqrt{3})a^2 + 4a - 7$
Here, the power of a are 3, 2 and 1 respectively
3, 2 and 1 are all whole numbers.
Hence, $(1/7)a^3 - (2/\sqrt{3})a^2 + 4a - 7$ is a polynomial.

(viii)

$$\frac{1}{2x}$$

$1/2x = (x^{-1}/2)$
Here, the power of x = – 1, which is not a whole number, but a negative number.
Hence, 1/2x is not a polynomial

EXERCISE 2.3

1. **Classify the following polynomials as polynomials in one variable, two variables etc.**

(i) $x^2 + x + 1$
(ii) $y^3 – 5y$
(iii) $xy + yz + zx$
(iv) $x^2 – 2xy + y^2 + 1$

Solution:

(i) $x^2 + x + 1$
Here, the polynomial contains only one variable, i.e., x.
Hence, the given polynomial is a polynomial in **one** variable.

(ii) $y^3 – 5y$
Here, the polynomial contains only one variable, i.e., y.
Hence, the given polynomial is a polynomial in **one** variable.

(iii) $xy + yz + zx$
Here, the polynomial contains three variables, i.e., x, y and z.
Hence, the given polynomial is a polynomial in **three** variable.

(iv) $x^2 – 2xy + y^2 + 1$
Here, the polynomial contains two variables, i.e., x and y.
Hence, the given polynomial is a polynomial in **two** variable.

2. Determine the degree of each of the following polynomials:

(i) $2x – 1$
(ii) $–10$
(iii) $x^3 – 9x + 3x^5$
(iv) $y^3 (1 – y^4)$

Solution:

Degree of a polynomial in one variable = highest power of the variable in algebraic expression
(i) $2x – 1$
Power of x = 1
Highest power of the variable x in the given expression = 1
Hence, degree of the polynomial $2x – 1 = 1$

(ii) $–10$
There is no variable in the given term.
Let us assume that the variable in the given expression is x.
$– 10 = –10x^0$
Power of x = 0
Highest power of the variable x in the given expression = 0
Hence, degree of the polynomial $– 10 = 0$

(iii) $x^3 – 9x + 3x^5$

Powers of x = 3, 1 and 5 respectively.
Highest power of the variable x in the given expression = 5
Hence, degree of the polynomial $x^3 - 9x + 3x^5 = 5$

(iv) $y^3 (1 - y^4)$
The equation can be written as,
$y^3 (1 - y^4) = y^3 - y^7$
Powers of y = 3 and 7 respectively.
Highest power of the variable y in the given expression = 7
Hence, degree of the polynomial $y^3 (1 - y^4) = 7$

3. For the polynomial
$\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$ **, write**
(i) the degree of the polynomial
(ii) the coefficient of x^3
(iii) the coefficient of x^6
(iv) the constant term
Solution:
The given polynomial is
$$\frac{(x^3 + 2x + 1)}{5} - \frac{7}{2}x^2 - x^6$$

(i)Powers of x = 3, 1, 2 and 6 respectively.
Highest power of the variable x in the given expression = 6
Hence, degree of the polynomial = 6

(ii) The given equation can be written as,
$$\frac{(x^3 + 2x + 1)}{5} - \frac{7}{2}x^2 - x^6 = \frac{1}{5}x^3 + \frac{2}{5}x + \frac{1}{5} - \frac{7}{2}x^2 - x^6$$
Hence, the coefficient of x^3 in the given polynomial is 1/5.

(iii) The coefficient of x^6 in the given polynomial is – 1

(iv) Since the given equation can be written as,
$$\frac{(x^3 + 2x + 1)}{5} - \frac{7}{2}x^2 - x^6 = \frac{1}{5}x^3 + \frac{2}{5}x + \frac{1}{5} - \frac{7}{2}x^2 - x^6$$
The constant term in the given polynomial is 1/5 as it has no variable x associated with it.

4. Write the coefficient of x^2 in each of the following:
(i) $(\pi/6)x + x^2 - 1$
(ii) $3x - 5$
(iii) $(x - 1)(3x - 4)$
(iv) $(2x - 5)(2x^2 - 3x + 1)$

Solution:

(i) $(\pi/6)\ x + x^2-1$

$(\pi/6)\ x + x^2-1 = (\pi/6)\ x + (1)\ x^2-1$

The coefficient of x^2 in the polynomial $(\pi/6)\ x + x^2-1 = 1$.

(ii) $3x - 5$

$3x - 5 = 0x^2 + 3x - 5$

The coefficient of x^2 in the polynomial $3x - 5 = 0$, zero.

(iii) $(x - 1)\ (3x - 4)$

$(x - 1)(3x - 4) = 3x^2 - 4x - 3x + 4$

$= 3x^2 - 7x + 4$

The coefficient of x^2 in the polynomial $3x^2 - 7x + 4 = 3$.

(iv) $(2x - 5)\ (2x^2 - 3x + 1)$

$(2x - 5)\ (2x^2 - 3x + 1)$

$= 4x^3 - 6x^2 + 2x - 10x^2 + 15x - 5$

$= 4x^3 - 16x^2 + 17x - 5$

The coefficient of x^2 in the polynomial $(2x - 5)\ (2x^2 - 3x + 1) = -16$

5. Classify the following as a constant, linear, quadratic and cubic polynomials:

(i) $2 - x^2 + x^3$

(ii) $3x^3$

(iii) $5t - \sqrt{7}$

(iv) $4 - 5y^2$

(v) 3

(vi) $2 + x$

(vii) $y^3 - y$

(viii) $1 + x + x^2$

(ix) t^2

(x) $\sqrt{2}x - 1$

Solution:

Constant polynomials: The polynomial of the degree zero.

Linear polynomials: The polynomial of degree one.

Quadratic polynomials: The polynomial of degree two.

Cubic polynomials: The polynomial of degree three.

(i) $2 - x^2 + x^3$

Powers of x = 2, and 3 respectively.

Highest power of the variable x in the given expression = 3

Hence, degree of the polynomial = 3

Since it is a polynomial of the degree 3, it is a cubic polynomial.

(ii) $3x^3$

Power of x = 3.

Highest power of the variable x in the given expression = 3

Hence, degree of the polynomial = 3

Since it is a polynomial of the degree 3, it is a cubic polynomial.

(iii) $5t - \sqrt{7}$

Power of t = 1.
Highest power of the variable t in the given expression = 1
Hence, degree of the polynomial = 1
Since it is a polynomial of the degree 1, it is a linear polynomial.

(iv) $4 - 5y^2$

Power of y = 2.
Highest power of the variable y in the given expression = 2
Hence, degree of the polynomial = 2
Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(v) 3

There is no variable in the given expression.
Let us assume that x is the variable in the given expression.
3 can be written as $3x^0$.
i.e., $3 = x^0$
Power of x = 0.
Highest power of the variable x in the given expression = 0
Hence, degree of the polynomial = 0
Since it is a polynomial of the degree 0, it is a constant polynomial.

(vi) $2 + x$

Power of x = 1.
Highest power of the variable x in the given expression = 1
Hence, degree of the polynomial = 1
Since it is a polynomial of the degree 1, it is a linear polynomial.

(vii) $y^3 - y$

Powers of y = 3 and 1, respectively.
Highest power of the variable x in the given expression = 3
Hence, degree of the polynomial = 3
Since it is a polynomial of the degree 3, it is a cubic polynomial.

(viii) $1 + x + x^2$

Powers of x = 1 and 2, respectively.
Highest power of the variable x in the given expression = 2
Hence, degree of the polynomial = 2
Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(ix) t^2

Power of t = 2.
Highest power of the variable t in the given expression = 2
Hence, degree of the polynomial = 2

Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(x) √2x – 1

Power of x = 1.

Highest power of the variable x in the given expression = 1

Hence, degree of the polynomial = 1

Since it is a polynomial of the degree 1, it is a linear polynomial.

6. Give an example of a polynomial, which is:

(i) monomial of degree 1

(ii) binomial of degree 20

(iii) trinomial of degree 2

Solution:

(i) Monomial = an algebraic expression that contains one term

An example of a polynomial, which is a monomial of degree 1 = 2t

(ii) Binomial = an algebraic expression that contains two terms

An example of a polynomial, which is a binomial of degree 20 = $x^{20} + 5$

(iii) Trinomial = an algebraic expression that contains three terms

An example of a polynomial, which is a trinomial of degree 2 = $y^2 + 3y + 11$

7. Find the value of the polynomial $3x^3 – 4x^2 + 7x – 5$, when x = 3 and also when x = –3.

Solution:

Given that,

$p(x) = 3x^3 – 4x^2 + 7x – 5$

According to the question,

<u>When x = 3,</u>

p(x) = p(3)

$p(x) = 3x^3 – 4x^2 + 7x – 5$

Substituting x = 3,

$p(3)= 3(3)^3 – 4(3)^2 + 7(3) – 5$

$p(3) = 3(3)^3 – 4(3)^2 + 7(3) – 5$

$= 3(27) – 4(9) + 21 – 5$

$= 81 – 36 + 21 – 5$

$= 102 – 41$

$= 61$

<u>When x = – 3,</u>

p(x) = p(– 3)

$p(x) = 3x^3 – 4x^2 + 7x – 5$

Substituting x = – 3,

$p(– 3)= 3(– 3)^3 – 4(– 3)^2 + 7(– 3) – 5$

$p(– 3) = 3(–3)^3 – 4(–3)^2 + 7(–3) – 5$

$= 3(–27) – 4(9) – 21 – 5$

$= –81 – 36 – 21 – 5$

$= –143$

8. If $p(x) = x^2 - 4x + 3$, evaluate: $p(2) - p(-1) + p(½)$.

Given that,

$p(x) = x^2 - 4x + 3$

According to the question,

When x = 2,

p(x) = p(2)

$p(x) = x^2 - 4x + 3$

Substituting x = 2,

$p(2) = (2)^2 - 4(2) + 3$
$= 4 - 8 + 3$
$= -4 + 3$
$= -1$

When x = – 1,

p(x) = p(– 1)

$p(x) = x^2 - 4x + 3$

Substituting x = – 1,

$p(-1) = (-1)^2 - 4(-1) + 3$
$= 1 + 4 + 3$
$= 8$

When x = ½ ,

p(x) = p(½)

$p(x) = x^2 - 4x + 3$

Substituting x = ½,

$p(½) = (½)^2 - 4(½) + 3$
$= ¼ - 2 + 3$
$= ¼ + 1$
$= 5/4$

Now,

$p(2) - p(-1) + p(½) = -1 - 8 + (5/4)$
$= -9 + (5/4)$
$= (-36 + 5)/4$
$= -31/4$

9. Find p(0), p(1), $p(-2)$ for the following polynomials:

(i) $(x) = 10x - 4x^2 - 3$

(ii) $(y) = (y + 2)(y - 2)$

Solution:

(i) According to the question,

$p(x) = 10x - 4x^2 - 3$

When x = 0,

p(x) = p(0)

Substituting x = 0,

$p(0) = 10(0) - 4(0)^2 - 3$
$= 0 - 0 - 3$

$= -3$

<u>When x = 1,</u>

p(x) = p(1)

Substituting x = 1,

$p(1) = 10(1) - 4(1)^2 - 3$

$= 10 - 4 - 3$

$= 6 - 3$

$= 3$

<u>When x = – 2,</u>

p(x) = p(– 2)

Substituting x = – 2,

$p(-2) = 10(-2) - 4(-2)^2 - 3$

$= -20 - 16 - 3$

$= -36 - 3$

$= -39$

(ii) According to the question,

$p(y) = (y + 2)(y - 2)$

<u>When y = 0,</u>

p(y) = p(0)

Substituting y = 0,

$p(0) = (0 + 2)(0 - 2)$

$= (2)(-2)$

$= -4$

<u>When y = 1,</u>

p(y) = p(1)

Substituting y = 1,

$p(1) = (1 + 2)(1 - 2)$

$= (3)(-1)$

$= -3$

<u>When y = – 2,</u>

p(y) = p(– 2)

Substituting y = – 2,

$p(-2) = (-2 + 2)(-2 - 2)$

$= (0)(-4)$

$= 0$

10. Verify whether the following are true or false:

(i) –3 is a zero of x – 3

(ii) – 1/3 is a zero of 3x + 1

(iii) – 4/5 is a zero of 4 –5y

(iv) 0 and 2 are the zeroes of $t^2 – 2t$
(v) –3 is a zero of $y^2 + y – 6$

Solution:

(i) –3 is a zero of x – 3

False

Zero of x – 3 is given by,

$x – 3 = 0$

$\Rightarrow x=3$

(ii) – 1/3 is a zero of 3x + 1

True

Zero of 3x + 1 is given by,

$3x + 1 = 0$

$\Rightarrow 3x = – 1$

$\Rightarrow x = – 1/3$

(iii) – 4/5 is a zero of 4 –5y

False

Zero of 4 – 5y is given by,

$4 – 5y =0$

$\Rightarrow – 5y = – 4$

$\Rightarrow y = 4/5$

(iv) 0 and 2 are the zeroes of $t^2 – 2t$

True

Zeros of $t^2 – 2t$ is given by,

$t^2 – 2t = t(t – 2) = 0$

$\Rightarrow t = 0$ or 2

(v) –3 is a zero of $y^2 + y – 6$

True

Zero of $y^2 + y – 6$ is given by,

$y^2 + y – 6 = 0$

$\Rightarrow y^2 + 3x – 2x – 6 = 0$

$\Rightarrow y (y + 3) – 2(x + 3) = 0$

$\Rightarrow (y – 2) (y + 3) =0$

$\Rightarrow y = 2$ or $– 3$

11. Find the zeroes of the polynomial in each of the following:

(i) p(x) = x – 4
(ii) g(x) = 3 – 6x
(iii) q(x) = 2x –7
(iv) h(y) = 2y

Solution:

(i) p(x) = x – 4

Zero of the polynomial $p(x) \Rightarrow p(x) = 0$

$P(x) = 0$
$\Rightarrow x - 4 = 0$
$\Rightarrow x = 4$
Therefore, the zero of the polynomial is 4.

(ii) $g(x) = 3 - 6x$
Zero of the polynomial $g(x) \Rightarrow g(x) = 0$
$g(x) = 0$
$\Rightarrow 3 - 6x = 0$
$\Rightarrow x = 3/6 = ½$
Therefore, the zero of the polynomial is ½

(iii) $q(x) = 2x - 7$
Zero of the polynomial $q(x) \Rightarrow q(x) = 0$
$q(x) = 0$
$\Rightarrow 2x - 7 = 0$
$\Rightarrow x = 7/2$
Therefore, the zero of the polynomial is 7/2

(iv) $h(y) = 2y$
Zero of the polynomial $h(y) \Rightarrow h(y) = 0$
$h(y) = 0$
$\Rightarrow 2y = 0$
$\Rightarrow y = 0$
Therefore, the zero of the polynomial is 0

12. Find the zeroes of the polynomial:
$p(x) = (x - 2)^2 - (x + 2)^2$
Solution:
$p(x) = (x - 2)^2 - (x + 2)^2$
We know that,
Zero of the polynomial $p(x) = 0$
Hence, we get,
$\Rightarrow (x - 2)^2 - (x + 2)^2 = 0$
Expanding using the identity, $a^2 - b^2 = (a - b)(a + b)$
$\Rightarrow (x - 2 + x + 2)(x - 2 - x - 2) = 0$
$\Rightarrow 2x(-4) = 0$
$\Rightarrow -8x = 0$
Therefore, the zero of the polynomial = 0

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial: $x^4 + 1$; $x - 1$
Solution:
Performing the long division method, we get,

$$
\begin{array}{l}
x-1 \overline{)\, x^4 + 1\,} \left(x^3 + x^2 + x + 1\right. \\
\quad x^4 - x^3 \\
\quad - \quad + \\
\hline
\quad x^3 + 1 \\
\quad x^3 - x^2 \\
\quad - \quad + \\
\hline
\qquad x^2 + 1 \\
\qquad x^2 - x \\
\qquad - \quad + \\
\hline
\qquad\quad x + 1 \\
\qquad\quad x - 1 \\
\qquad\quad - \quad + \\
\hline
\qquad\qquad 2
\end{array}
$$

Hence, from the above long division method, we get,
Quotient = $x^3 + x^2 + x + 1$
Remainder = 2.

14. By Remainder Theorem find the remainder, when p(x) is divided by g(x), where
(i) $p(x) = x^3 - 2x^2 - 4x - 1$, $g(x) = x + 1$
(ii) $p(x) = x^3 - 3x^2 + 4x + 50$, $g(x) = x - 3$
(iii) $p(x) = 4x^3 - 12x^2 + 14x - 3$, $g(x) = 2x - 1$
(iv) $p(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - 3/2\, x$
Solution:

(i) Given $p(x) = x^3 - 2x^2 - 4x - 1$ and $g(x) = x + 1$
Here zero of $g(x) = -1$
By using the remainder theorem
P(x) divided by g(x) = p(– 1)
$P(-1) = (-1)^3 - 2(-1)^2 - 4(-1) - 1 = 0$
Therefore, the remainder = 0

(ii) given $p(x) = x^3 - 3x^2 + 4x + 50$, $g(x) = x - 3$
Here zero of g(x) = 3
By using the remainder theorem p(x) divided by g(x) = p(3)
$p(3) = 3^3 - 3 \times (3)^2 + 4 \times 3 + 50 = 62$
Therefore, the remainder = 62

(iii) $p(x) = 4x^3 - 12x^2 + 14x - 3$, $g(x) = 2x - 1$
Here zero of g(x) = ½
By using the remainder theorem p(x) divided by g(x) = p (½)
$P(\tfrac12) = 4(\tfrac12)^3 - 12(\tfrac12)^2 + 14(\tfrac12) - 3$
$= 4/8 - 12/4 + 14/2 - 3$
$= \tfrac12 + 1$
$= 3/2$
Therefore, the remainder = 3/2

(iv) $p(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - 3/2\ x$

Here zero of g(x) = 2/3

By using the remainder theorem p(x) divided by g(x) = p(2/3)

$p(2/3) = (2/3)^3 - 6(2/3)^2 + 2(2/3) - 4$

$= -136/27$

Therefore, the remainder = – 136/27

15. Check whether p(x) is a multiple of g(x) or not:

(i) $p(x) = x^3 - 5x^2 + 4x - 3$, $g(x) = x - 2$

(ii) $p(x)= 2x^3 - 11x^2 - 4x + 5$, $g(x)= 2x + 1$

Solution:

(i)

According to the question,

$g(x)=x - 2$,

Then, zero of g(x),

$g(x) = 0$

$x - 2 = 0$

$x = 2$

Therefore, zero of g(x) = 2

So, substituting the value of x in p(x), we get,

$p(2) = (2)^3 - 5(2)^2 + 4(2) - 3$

$= 8 - 20 + 8 - 3$

$= -7 \neq 0$

Hence, p(x) is not the multiple of g(x) since the remainder ≠ 0.

(ii)

According to the question,

$g(x)= 2x + 1$

Then, zero of g(x),

$g(x) = 0$

$2x + 1 = 0$

$2x = -1$

$x = -½$

Therefore, zero of g(x) = – ½

So, substituting the value of x in p(x), we get,

$p(-½) = 2 \times (-½)^3 - 11 \times (-½)^2 - 4 \times (-1/2) + 5$

$= -¼ - 11/4 + 7$

$= 16/4$

$= 4 \neq 0$

Hence, p(x) is not the multiple of g(x) since the remainder ≠ 0.

16. Show that:

(i) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$.

(ii) $2x-3$ is a factor of $x + 2x^3 - 9x^2 + 12$

Solution:

(i)According to the question,

Let $p(x) = 69 + 11x - x^2 + x^3$ and $g(x) = x + 3$
$g(x) = x + 3$
zero of $g(x) \Rightarrow g(x) = 0$
$x + 3 = 0$
$x = -3$
Therefore, zero of $g(x) = -3$
So, substituting the value of x in p(x), we get,
$p(-3) = 69 + 11(-3) - (-3)^2 + (-3)^3$
$= 69 - 69$
$= 0$
Since, the remainder = zero,
We can say that,
$g(x) = x + 3$ is factor of $p(x) = 69 + 11x - x^2 + x^3$

(ii) According to the question,
Let $p(x) = x + 2x^3 - 9x^2 + 12$ and $g(x) = 2x - 3$
$g(x) = 2x - 3$
zero of $g(x) \Rightarrow g(x) = 0$
$2x - 3 = 0$
$x = 3/2$
Therefore, zero of $g(x) = 3/2$
So, substituting the value of x in p(x), we get,
$P(3/2) = 3/2 + 2(3/2)^3 - 9(3/2)^2 + 12$
$= (81 - 81) / 4$
$= 0$
Since, the remainder = zero,
We can say that,
$g(x) = 2x - 3$ is factor of $p(x) = x + 2x^3 - 9x^2 + 12$

17. Determine which of the following polynomials has x – 2 a factor:
(i) $3x^2 + 6x - 24$.
(ii) $4x^2 + x - 2$.

Solution:
(i) According to the question,
Let $p(x) = 3x^2 + 6x - 24$ and $g(x) = x - 2$
$g(x) = x - 2$
zero of $g(x) \Rightarrow g(x) = 0$
$x - 2 = 0$
$x = 2$
Therefore, zero of $g(x) = 2$
So, substituting the value of x in p(x), we get,
$p(2) = 3(2)^2 + 6(2) - 24$
$= 12 + 12 - 24$
$= 0$
Since, the remainder = zero,
We can say that,

$g(x) = x – 2$ is factor of $p(x) = 3x^2 + 6x – 24$

(ii) According to the question,
Let $p(x) = 4x^2 + x – 2$ and $g(x) = x – 2$
$g(x) = x – 2$
zero of $g(x) \Rightarrow g(x) = 0$
$x – 2 = 0$
$x = 2$
Therefore, zero of $g(x) = 2$
So, substituting the value of x in p(x), we get,
$p(2) \quad = 4(2)^2 + 2 – 2$
$\quad\quad = 16 \neq 0$
Since, the remainder $\neq$ zero,
We can say that,
$g(x) = x – 2$ is not a factor of $p(x) = 4x^2 + x - 2$

18. Show that p – 1 is a factor of $p^{10} – 1$ and also of $p^{11} – 1$.
Solution:
According to the question,
Let $h(p) = p^{10} – 1$,and $g(p) = p – 1$
zero of $g(p) \Rightarrow g(p) = 0$
$p – 1 = 0$
$p = 1$
Therefore, zero of $g(x) = 1$
We know that,
According to factor theorem if g(p) is a factor of h(p) , then h(1) should be zero
So,
$h(1) = (1)^{10} – 1 = 1 − 1 = 0$
$\Longrightarrow$ g (p) is a factor of h(p).
Now, we have $h(p) = p^{11} − 1$, $g(p) = p – 1$
Putting $g(p) = 0 \Longrightarrow p − 1 = 0 \Longrightarrow p = 1$
According to factor theorem if g (p) is a factor of h(p) ,
Then $h(1) = 0$
$\Longrightarrow (1)^{11} – 1 = 0$
Therefore, $g(p) = p – 1$ is the factor of $h(p) = p^{10} – 1$

19. For what value of m is $x^3 – 2mx^2 + 16$ divisible by x + 2?
Solution:
According to the question,
Let $p(x) = x^3 – 2mx^2 + 16$, and $g(x) = x + 2$
$g(x) = 0$
$\Longrightarrow x + 2 = 0$
$\Longrightarrow x = – 2$
Therefore, zero of $g(x) = – 2$
We know that,
According to factor theorem,

if p(x) is divisible by g(x), then the remainder p(−2) should be zero.
So, substituting the value of x in p(x), we get,
$p(-2) = 0$
$\Rightarrow (-2)^3 - 2m(-2)^2 + 16 = 0$
$\Rightarrow 0 - 8 - 8m + 16 = 0$
$\Rightarrow 8m = 8$
$\Rightarrow m = 1$

20. If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a.
Solution:
According to the question,
Let $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$ and $g(x) = x + 2a$
$g(x) = 0$
$\Rightarrow x + 2a = 0$
$\Rightarrow x = -2a$
Therefore, zero of $g(x) = -2a$
We know that,
According to the factor theorem,
If g(x) is a factor of p(x), then $p(-2a) = 0$
So, substituting the value of x in p(x), we get,
$p(-2a) = (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3 = 0$
$\Rightarrow -32a^5 + 32a^5 - 2a + 3 = 0$
$\Rightarrow -2a = -3$
$\Rightarrow a = 3/2$

EXERCISE 2.4

1. If the polynomials $az^3 + 4z^2 + 3z – 4$ and $z^3 – 4z + a$ leave the same remainder when divided by $z – 3$, find the value of a.

Solution:

Zero of the polynomial,
$g_1(z) = 0$
$z-3 = 0$
$z = 3$
Therefore, zero of g(z) = – 2a
Let $p(z) = az^3+4z^2+3z-4$
So, substituting the value of z = 3 in p(z), we get,
$p(3) = a(3)^3+4(3)^2+3(3)-4$
$\Rightarrow p(3) = 27a+36+9-4$
$\Rightarrow p(3) = 27a+41$
Let $h(z) = z^3-4z+a$
So, substituting the value of z = 3 in h(z), we get,
$h(3) = (3)^3-4(3)+a$
$\Rightarrow h(3) = 27-12+a$
$\Rightarrow h(3) = 15+a$
According to the question,
We know that,
The two polynomials, p(z) and h(z), leaves same remainder when divided by z-3
So, h(3)=p(3)
$\Rightarrow 15+a = 27a+41$
$\Rightarrow 15-41 = 27a - a$
$\Rightarrow -26 = 26a$
$\Rightarrow a = -1$

2. The polynomial $p(x) = x^4 – 2x^3 + 3x^2 – ax + 3a – 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a. Also find the remainder when $p(x)$ is divided by $x + 2$.

Solution:

$p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7.$
Divisor = x + 1
x + 1 = 0
x = -1
So, substituting the value of x = – 1 in p(x), we get,
$p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7.$
19 = 1 + 2 + 3 + a + 3a – 7
19 = 6 - 7 + 4a
4a - 1 = 19
4a = 20
a = 5
Since, a = 5.
We get the polynomial,

$p(x) = x^4 - 2x^3 + 3x^2 - (5)x + 3(5) – 7$
$p(x) = x^4 - 2x^3 + 3x^2 - 5x + 15 – 7$
$p(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$
As per the question,
When the polynomial obtained is divided by (x + 2),
We get,
$x + 2 = 0$
$x = - 2$
So, substituting the value of $x = – 2$ in p(x), we get,
$p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 5(-2) + 8$
$\Rightarrow p(-2) = 16 + 16 + 12 + 10 + 8$
$\Rightarrow p(-2) = 62$
Therefore, the remainder = 62.

3. If both $x – 2$ and $x – ½$ are factors of $px^2 + 5x + r$, show that $p = r$.
Solution:
Given, $f(x) = px^2+5x+r$ and factors are $x-2$, $x - ½$
$g_1(x) = 0$,
$x – 2 = 0$
$x = 2$
Substituting $x = 2$ in place of equation, we get
$f(x) = px^2+5x+r$
$f(2) \quad = p(2)^2+5(2)+r=0$
$\quad = 4p + 10 + r = 0 \quad \dots$ eq.(i)
$x – ½ = 0$
$x = ½$
Substituting $x = ½$ in place of equation, we get,
$f(x) = px^2+5x+r$
$f(½) \quad = p(½)^2 + 5(½) + r =0$
$\quad = p/4 + 5/2 + r = 0$
$\quad = p + 10 + 4r = 0 \quad \dots$ eq(ii)
On solving eq(i) and eq(ii),
We get,
$4p + r = – 10$ and $p + 4r = – 10$
Since the RHS of both the equations are same,
We get,
$4p + r = p + 4r$
$3p=3r$
$p = r$.
Hence Proved.

4. Without actual division, prove that $2x^4 – 5x^3 + 2x^2 – x + 2$ is divisible by $x^2 – 3x + 2$. [Hint: Factorise $x^2 – 3x + 2$]
Solution:
x^2-3x+2
$x^2-2x-1x+2$

$x(x-2)-1(x-2)$
$(x-2)(x-1)$
Therefore,$(x-2)(x-1)$are the factors.

Considering $(x-2)$,
$x-2=0$
$x=2$
Then, $p(x)$ becomes,
$p(x)=2$
$p(x)=2x^4-5x^3+2x^2-x+2$
$p(2)=2(2)^4-5(2)^3+2(2)^2-2+2$
$=32-40+8$
$= -40+40=0$
Therefore, $(x-2)$ is a factor.

Considering $(x-1)$,
$x-1=0$
$x=1$
Then, $p(x)$ becomes,
$p(x)=1$
$p(x)=2x^4-5x^3+2x^2-x+2$
$p(1)=2(1)^4-5(1)^3+2(1)^2-1+2$
$=2-5+2-1+2$
$=6-6$
$=0$
Therefore, $(x-1)$ is a factor.

Chapter-2

Polynomials

Exercise 2.1

Question 1:

Which of the following expressions are polynomials in one variable and which are not? State reasons for your **Answer**.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Answer:

(i) $4x^2 - 3x + 7$

Yes, this expression is a polynomial in one variable *x*.

(ii) $y^2 + \sqrt{2}$

Yes, this expression is a polynomial in one variable *y*.

(iii) $3\sqrt{t} + t\sqrt{2}$

No. It can be observed that the exponent of variable *t* in term $3\sqrt{t}$ is $\frac{1}{2}$, which is not a whole number. Therefore, this expression is not a polynomial.

(iv) $y + \frac{2}{y}$

No. It can be observed that the exponent of variable *y* in term $\frac{2}{y}$ is −1, which is not a whole number. Therefore, this expression is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$

No. It can be observed that this expression is a polynomial in 3 variables *x*, *y*, and *t*. Therefore, it is not a polynomial in one variable.

Question 2:

Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$ (ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$ (iv) $\sqrt{2}x-1$

Answer:

(i) $2+x^2+x$

In the above expression, the coefficient of x^2 is 1.

(ii) $2-x^2+x^3$

In the above expression, the coefficient of x^2 is −1.

(iii) $\frac{\pi}{2}x^2+x$

In the above expression, the coefficient of x^2 is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1$, or

$0.x^2+\sqrt{2}x-1$

In the above expression, the coefficient of x^2 is 0.

Question 3:

Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

Binomial has two terms in it. Therefore, binomial of degree 35 can be written as $x^{35}+x^{34}$. Monomial has only one term in it. Therefore, monomial of degree 100 can be written as x^{100}.

Question 4:

Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$ (ii) $4-y^2$

(iii) $5t-\sqrt{7}$ (iv) 3

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $5x^3+4x^2+7x$

This is a polynomial in variable *x* and the highest power of variable *x* is 3. Therefore, the degree of this polynomial is 3.

(ii) $4-y^2$

This is a polynomial in variable *y* and the highest power of variable *y* is 2. Therefore, the degree of this polynomial is 2.

(iii) $5t-\sqrt{7}$

This is a polynomial in variable *t* and the highest power of variable *t* is 1. Therefore, the degree of this polynomial is 1.

(iv) 3

This is a constant polynomial. Degree of a constant polynomial is always 0.

Question 5:

Classify the following as linear, quadratic and cubic polynomial:

(i) x^2+x (ii) $x-x^3$ (iii) $y+y^2+4$ (iv) $1+x$ (v) $3t$

(vi) r^2 (vii) $7x^3$

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively.

(i) $x^2 + x$ is a quadratic polynomial as its degree is 2.

(ii) $x - x^3$ is a cubic polynomial as its degree is 3.

(iii) $y + y^2 + 4$ is a quadratic polynomial as its degree is 2.

(iv) 1 + x is a linear polynomial as its degree is 1.

(v) $3t$ is a linear polynomial as its degree is 1.

(vi) r^2 is a quadratic polynomial as its degree is 2.

(vii) $7x^3$ is a cubic polynomial as its degree is 3.

Exercise 2.2

Question 1:

Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Answer:

(i) $p(x) = 5x - 4x^2 + 3$

$$p(0) = 5(0) - 4(0)^2 + 3$$
$$= 3$$

(ii) $p(x) = 5x - 4x^2 + 3$

$$p(-1) = 5(-1) - 4(-1)^2 + 3$$
$$= -5 - 4(1) + 3 = -6$$

(iii) $p(x) = 5x - 4x^2 + 3$

$$p(2) = 5(2) - 4(2)^2 + 3$$
$$= 10 - 16 + 3 = -3$$

Question 2:

Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Answer:

(i) $p(y) = y^2 - y + 1$

$p(0) = (0)^2 - (0) + 1 = 1$

$p(1) = (1)^2 - (1) + 1 = 1$

$p(2) = (2)^2 - (2) + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$

$p(1) = 2 + (1) + 2(1)^2 - (1)^3$

$= 2 + 1 + 2 - 1 = 4$

$p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

$p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$

$p(2) = (2)^3 = 8$

(iv) $p(x) = (x - 1)(x + 1)$

$p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$

$p(1) = (1 - 1)(1 + 1) = 0(2) = 0$

$p(2) = (2 - 1)(2 + 1) = 1(3) = 3$

Question 3:

Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1, x = 1, -1$ (iv) $p(x) = (x + 1)(x - 2), x = -1, 2$

(v) $p(x) = x^2, x = 0$ (vi) $p(x) = lx + m$

(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ (viii) $p(x) = 2x + 1, x = \frac{1}{2}$

Answer:

(i) If $x = \frac{-1}{3}$ is a zero of given polynomial $p(x) = 3x + 1$, then $p\left(-\frac{1}{3}\right)$ should be 0.

$$\text{Here, } p\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right) + 1 = -1 + 1 = 0$$

Therefore, $x = \frac{-1}{3}$ is a zero of the given polynomial.

(ii) If $x = \frac{4}{5}$ is a zero of polynomial $p(x) = 5x - \pi$, then $p\left(\frac{4}{5}\right)$ should be 0.

$$Here,\ p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi$$

$$As\ p\left(\frac{4}{5}\right) \neq 0,$$

Therefore, $x = \frac{4}{5}$ is not a zero of the given polynomial.

(iii) If $x = 1$ and $x = -1$ are zeroes of polynomial $p(x) = x^2 - 1$, then $p(1)$ and $p(-1)$ should be 0.

Here, $p(1) = (1)^2 - 1 = 0$, and

$p(-1) = (-1)^2 - 1 = 0$

Hence, $x = 1$ and -1 are zeroes of the given polynomial.

(iv) If $x = -1$ and $x = 2$ are zeroes of polynomial $p(x) = (x+1)(x-2)$, then $p(-1)$ and $p(2)$should be 0.

Here, $p(-1) = (-1+1)(-1-2) = 0(-3) = 0$, and

$p(2) = (2+1)(2-2) = 3(0) = 0$

Therefore, $x = -1$ and $x = 2$ are zeroes of the given polynomial.

(v) If $x = 0$ is a zero of polynomial $p(x) = x^2$, then $p(0)$ should be zero.

Here, $p(0) = (0)^2 = 0$

Hence, $x = 0$ is a zero of the given polynomial.

(vi) If $x = \frac{-m}{l}$ is a zero of polynomial $p(x) = lx + m$, then $p\left(\frac{-m}{l}\right)$ should be 0.

Here, $p\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = 0$

Therefore, $x = -\frac{m}{l}$ is a zero of the given polynomial.

(vii) If $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 - 1$, then

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

Here, $p\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$, and

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$

Hence, $x = \frac{-1}{\sqrt{3}}$ is a zero of the given polynomial. However, $x = \frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

(viii) If $x = \frac{1}{2}$ is a zero of polynomial $p(x) = 2x + 1$, then $p\left(\frac{1}{2}\right)$ should be 0.

Here, $p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)+1=1+1=2$

As $p\left(\frac{1}{2}\right)\neq 0$,

Therefore, $x=\frac{1}{2}$ is not a zero of the given polynomial.

Question 4:

Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$ (ii) $p(x) = x - 5$ (iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$ (v) $p(x) = 3x$ (vi) $p(x) = ax$, $a \neq 0$

(vii) $p(x) = cx + d$, $c \neq 0$, c, are real numbers.

Answer:

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

(i) $p(x) = x + 5$

$p(x) = 0$

$x + 5 = 0$

$x = -5$

Therefore, for $x = -5$, the value of the polynomial is 0 and hence, $x = -5$ is a zero of the given polynomial.

(ii) $p(x) = x - 5$

$p(x) = 0$

$x - 5 = 0$

$x = 5$

Therefore, for $x = 5$, the value of the polynomial is0 and hence, $x = 5$ is a zero of the given polynomial.

(iii) $p(x) = 2x + 5$

$p(x) = 0$

$2x + 5 = 0$

$2x = -5$

$$x = -\frac{5}{2}$$

Therefore, for $x = -\frac{5}{2}$, the value of the polynomial is 0 and hence, $x = \frac{-5}{2}$ is a zero of the given polynomial.

(iv) $p(x) = 3x - 2$

$p(x) = 0$

$3x - 2 = 0$

$$x = \frac{2}{3}$$

Therefore, for $x = \frac{2}{3}$, the value of the polynomial is 0 and hence, $x = \frac{2}{3}$ is a zero of the given polynomial.

(v) $p(x) = 3x$

$p(x) = 0$

$3x = 0$

$x = 0$

Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vi) $p(x) = ax$

$p(x) = 0$

$ax = 0$

$x = 0$

Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vii) $p(x) = cx + d$

$p(x) = 0$

$cx + d = 0$

$x = \frac{-d}{c}$

Therefore, for $x = \frac{-d}{c}$, the value of the polynomial is 0 and hence, $x = \frac{-d}{c}$ is a zero of the given polynomial.

Exercise 2.3

Question 1:

Determine which of the following polynomials has $(x + 1)$ a factor:

(i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ (iv) $x^3 - x^2 - \left(2 + \sqrt{2}\right)x + \sqrt{2}$

Answer:

(i) If $(x + 1)$ is a factor of $p(x) = x^3 + x^2 + x + 1$, then $p(-1)$ must be zero, otherwise $(x + 1)$ is not a factor of $p(x)$.

$p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

Hence, $x + 1$ is a factor of this polynomial.

(ii) If $(x + 1)$ is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$, then $p(-1)$ must be zero, otherwise $(x + 1)$ is not a factor of $p(x)$.

$p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1 = 1$

As $p(-1) \neq 0$,

Therefore, $x + 1$ is not a factor of this polynomial.

(iii) If $(x + 1)$ is a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$, then $p(-1)$ must be 0, otherwise $(x + 1)$ is not a factor of this polynomial.

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

As $p(-1) \neq 0$,

Therefore, $x + 1$ is not a factor of this polynomial.

(iv) If$(x + 1)$ is a factor of polynomial $p(x) = x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$, then $p(-1)$ must be 0, otherwise $(x + 1)$ is not a factor of this polynomial.

$$
\begin{aligned}
p(-1) &= (-1)^3 - (-1)^2 - \left(2+\sqrt{2}\right)(-1) + \sqrt{2} \\
&= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} \\
&= 2\sqrt{2}
\end{aligned}
$$

As $p(-1) \neq 0$,

Therefore, $(x + 1)$ is not a factor of this polynomial.

Question 2:

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Answer:

(i) If $g(x) = x + 1$ is a factor of the given polynomial $p(x)$, then $p(-1)$ must be zero.

$p(x) = 2x^3 + x^2 - 2x - 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2(-1) + 1 + 2 - 1 = 0$

Hence, $g(x) = x + 1$ is a factor of the given polynomial.

(ii) If $g(x) = x + 2$ is a factor of the given polynomial $p(x)$, then $p(-2)$ must be 0.

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1$

As $p(-2) \neq 0$,

Hence, $g(x) = x + 2$ is not a factor of the given polynomial.

(iii) If $g(x) = x - 3$ is a factor of the given polynomial $p(x)$, then $p(3)$ must be 0.

$p(x) = x^3 - 4x^2 + x + 6$

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9 = 0$

Hence, $g(x) = x - 3$ is a factor of the given polynomial.

Question 3:

Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ (iv) $p(x) = kx^2 - 3x + k$

Answer:

If $x - 1$ is a factor of polynomial $p(x)$, then $p(1)$ must be 0.

(i) $p(x) = x^2 + x + k$

$p(1) = 0$

$\Rightarrow (1)^2 + 1 + k = 0$

$\Rightarrow 2 + k = 0 \Rightarrow k = -2$

Therefore, the value of k is −2.

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

$p(1) = 0$

$$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$$
$$\Rightarrow 2 + k + \sqrt{2} = 0$$
$$\Rightarrow k = -2 - \sqrt{2} = -\left(2 + \sqrt{2}\right)$$

Therefore, the value of k is $-\left(2+\sqrt{2}\right)$.

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

$p(1) = 0$

$$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$$
$$\Rightarrow k - \sqrt{2} + 1 = 0$$
$$\Rightarrow k = \sqrt{2} - 1$$

Therefore, the value of k is $\sqrt{2} - 1$.

(iv) $p(x) = kx^2 - 3x + k$

$\Rightarrow p(1) = 0$

$\Rightarrow k(1)^2 - 3(1) + k = 0$

$\Rightarrow k - 3 + k = 0$

$\Rightarrow 2k - 3 = 0$

$$\Rightarrow k = \frac{3}{2}$$

Therefore, the value of k is $\frac{3}{2}$.

Question 4:

Factorise:

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

Answer:

(i) $12x^2 - 7x + 1$

We can find two numbers such that $pq = 12 \times 1 = 12$ and $p + q = -7$. They are $p = -4$ and $q = -3$.

Here, $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$

(ii) $2x^2 + 7x + 3$

We can find two numbers such that $pq = 2 \times 3 = 6$ and $p + q = 7$.

They are $p = 6$ and $q = 1$.

Here, $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6$

We can find two numbers such that $pq = -36$ and $p + q = 5$.

They are $p = 9$ and $q = -4$.

Here,

$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

(iv) $3x^2 - x - 4$

We can find two numbers such that $pq = 3 \times (-4) = -12$

and $p + q = -1$.

They are $p = -4$ and $q = 3$.

Here,

$3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

Question 5:

Factorise:

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 + 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

Answer:

(i) Let $p(x) = x^3 - 2x^2 - x + 2$

All the factors of 2 have to be considered. These are ± 1, ± 2.

By trial method,

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$= -1 - 2 + 1 + 2 = 0$

Therefore, $(x + 1)$ is factor of polynomial $p(x)$.

Let us find the quotient on dividing $x^3 - 2x^2 - x + 2$ by $x + 1$.

By long division,

Exercise 2.4

Question 1:

Use suitable identities to find the following products:

(i) $(x+4)(x+10)$ (ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$ (iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Answer:

(i) By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$\begin{aligned}(x+4)(x+10)&=x^2+(4+10)x+4\times10\\&=x^2+14x+40\end{aligned}$$

(ii) By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$\begin{aligned}(x+8)(x-10)&=x^2+(8-10)x+(8)(-10)\\&=x^2-2x-80\end{aligned}$$

(iii) $(3x+4)(3x-5)=9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)$

By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$\begin{aligned}9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)&=9\left[x^2+\left(\frac{4}{3}-\frac{5}{3}\right)x+\left(\frac{4}{3}\right)\left(-\frac{5}{3}\right)\right]\\&=9\left[x^2-\frac{1}{3}x-\frac{20}{9}\right]\\&=9x^2-3x-20\end{aligned}$$

(iv) By using the identity $(x+y)(x-y)=x^2-y^2$,

$$\begin{aligned}\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)&=\left(y^2\right)^2-\left(\frac{3}{2}\right)^2\\&=y^4-\frac{9}{4}\end{aligned}$$

(v) By using the identity $(x+y)(x-y)=x^2-y^2$,

$$\begin{aligned}(3-2x)(3+2x)&=(3)^2-(2x)^2\\&=9-4x^2\end{aligned}$$

Question 2:

Evaluate the following products without multiplying directly:

(i) 103 × 107 (ii) 95 × 96 (iii) 104 × 96

Answer:

(i) 103 × 107 = (100 + 3) (100 + 7)

= $(100)^2$ + (3 + 7) 100 + (3) (7)

[By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$, where

x = 100, a = 3, and b = 7]

= 10000 + 1000 + 21

= 11021

(ii) 95 × 96 = (100 − 5) (100 − 4)

= $(100)^2$ + (− 5 − 4) 100 + (− 5) (− 4)

[By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$, where

x = 100, a = −5, and b = −4]

= 10000 − 900 + 20

= 9120

(iii) 104 × 96 = (100 + 4) (100 − 4)

= $(100)^2 - (4)^2$ $[(x+y)(x-y)=x^2-y^2]$

= 10000 − 16

= 9984

Question 3:

Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Answer:

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x+y)(3x+y)$ $\quad\quad\left[x^2 + 2xy + y^2 = (x+y)^2\right]$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y-1)(2y-1)$ $\quad\quad\left[x^2 - 2xy + y^2 = (x-y)^2\right]$

(iii) $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$

$= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$ $\quad\quad\left[x^2 - y^2 = (x+y)(x-y)\right]$

Question 4:

Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$ (ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$ (iv) $(3a-7b-c)^2$

(v) $(-2x+5y-3z)^2$ (vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Answer:

It is known that,

$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) $(x+2y+4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $(-2x + 3y + 2z)^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) $(3a - 7b - c)^2$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

(v) $(-2x + 5y - 3z)^2$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2\left(\frac{1}{4}a\right)(1)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

Question 5:

Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Answer:

It is known that,

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

$$\begin{aligned}&=(2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)+2(2x)(-4z)\\&=(2x+3y-4z)^2\\&=(2x+3y-4z)(2x+3y-4z)\end{aligned}$$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$$\begin{aligned}&=\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\left(-\sqrt{2}x\right)(y)+2(y)\left(2\sqrt{2}z\right)+2\left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)\\&=\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2\\&=\left(-\sqrt{2}x+y+2\sqrt{2}z\right)\left(-\sqrt{2}x+y+2\sqrt{2}z\right)\end{aligned}$$

Question 6:

Write the following cubes in expanded form:

(i) $(2x+1)^3$ (ii) $(2a-3b)^3$

(iii) $\left[\frac{3}{2}x+1\right]^3$ (iv) $\left[x-\frac{2}{3}y\right]^3$

Answer:

It is known that,

$$\begin{aligned}&(a+b)^3 = a^3+b^3+3ab(a+b)\\&\text{and}\,(a-b)^3 = a^3-b^3-3ab(a-b)\end{aligned}$$

(i) $(2x+1)^3 = (2x)^3+(1)^3+3(2x)(1)(2x+1)$

$$= 8x^3 + 1 + 6x(2x+1)$$
$$= 8x^3 + 1 + 12x^2 + 6x$$
$$= 8x^3 + 12x^2 + 6x + 1$$

(ii) $(2a-3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

$$= 8a^3 - 27b^3 - 18ab(2a-3b)$$
$$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$$

(iii) $\left[\frac{3}{2}x+1\right]^3 = \left[\frac{3}{2}x\right]^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$$
$$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$$
$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$$

(vi) $\left[x-\frac{2}{3}y\right]^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$$
$$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$

Question 7:

Evaluate the following using suitable identities:

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

Answer:

It is known that,

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$\text{and } (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

(i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 300(99)$

$= 1000000 - 1 - 29700$

$= 970299$

(ii) $(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200$

$= 1061208$

(iii) $(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$

$= 1000000000 - 8 - 6000(998)$

$= 1000000000 - 8 - 5988000$

$= 1000000000 - 5988008$

$= 994011992$

Question 8:

Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer:

It is known that,

$(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

and $(a-b)^3 = a^3-b^3-3a^2b+3ab^2$

(i) $8a^3+b^3+12a^2b+6ab^2$

$$= (2a)^3+(b)^3+3(2a)^2b+3(2a)(b)^2$$
$$= (2a+b)^3$$
$$= (2a+b)(2a+b)(2a+b)$$

(ii) $8a^3-b^3-12a^2b+6ab^2$

$$= (2a)^3-(b)^3-3(2a)^2b+3(2a)(b)^2$$
$$= (2a-b)^3$$
$$= (2a-b)(2a-b)(2a-b)$$

(iii) $27-125a^3-135a+225a^2$

$$= (3)^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$$
$$= (3-5a)^3$$
$$= (3-5a)(3-5a)(3-5a)$$

(iv) $64a^3-27b^3-144a^2b+108ab^2$

$$= (4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$$
$$= (4a-3b)^3$$
$$= (4a-3b)(4a-3b)(4a-3b)$$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2$$
$$= \left(3p - \frac{1}{6}\right)^3$$
$$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$$

Question 9:

Verify:

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Answer:

(i) It is known that,

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

$$\begin{aligned} x^3 + y^3 &= (x+y)^3 - 3xy(x+y) \\ &= (x+y)\left[(x+y)^2 - 3xy\right] \\ &= (x+y)(x^2 + y^2 + 2xy - 3xy) \\ &= (x+y)(x^2 + y^2 - xy) \\ &= (x+y)(x^2 - xy + y^2) \end{aligned}$$

(ii) It is known that,

$$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$$

$$\begin{aligned} x^3 - y^3 &= (x-y)^3 + 3xy(x-y) \\ &= (x-y)\left[(x-y)^2 + 3xy\right] \\ &= (x-y)\left(x^2 + y^2 - 2xy + 3xy\right) \\ &= (x-y)\left(x^2 + y^2 + xy\right) \\ &= (x-y)\left(x^2 + xy + y^2\right) \end{aligned}$$

Question 10:

Factorise each of the following:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

[**Hint:** See **Question** 9.]

Answer:

(i) $27y^3 + 125z^3$

$$\begin{aligned} &= (3y)^3 + (5z)^3 \\ &= (3y+5z)\left[(3y)^2 + (5z)^2 - (3y)(5z)\right] \qquad \left[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)\right] \\ &= (3y+5z)\left[9y^2 + 25z^2 - 15yz\right] \end{aligned}$$

(ii) $64m^3 - 343n^3$

$$\begin{aligned} &= (4m)^3 - (7n)^3 \\ &= (4m-7n)\left[(4m)^2 + (7n)^2 + (4m)(7n)\right] \qquad \left[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right] \\ &= (4m-7n)\left[16m^2 + 49n^2 + 28mn\right] \end{aligned}$$

Question 11:

Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Answer:

It is known that,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

$$\begin{aligned}
&\therefore 27x^3+y^3+z^3-9xyz\\
&=(3x)^3+(y)^3+(z)^3-3(3x)(y)(z)\\
&=(3x+y+z)\left[(3x)^2+y^2+z^2-(3x)(y)-(y)(z)-z(3x)\right]\\
&=(3x+y+z)\left[9x^2+y^2+z^2-3xy-yz-3xz\right]
\end{aligned}$$

Question 12:

Verify that $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

Answer:

It is known that,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

$$\begin{aligned}
&=\frac{1}{2}(x+y+z)\left[2x^2+2y^2+2z^2-2xy-2yz-2zx\right]\\
&=\frac{1}{2}(x+y+z)\left[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2zx)\right]\\
&=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]
\end{aligned}$$

Question 13:

If $x + y + z = 0$, show that

x3 + y3 + z3 = 3xyz.

Answer:

It is known that,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

Put $x + y + z = 0$,

$$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - zx)$$
$$x^3 + y^3 + z^3 - 3xyz = 0$$
$$x^3 + y^3 + z^3 = 3xyz$$

Question 14:

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Answer:

(i) $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12$, $y = 7$, and $z = 5$

It can be observed that,

$x + y + z = -12 + 7 + 5 = 0$

It is known that if $x + y + z = 0$, then

$$x^3 + y^3 + z^3 = 3xyz$$

$\therefore (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$

$= -1260$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $x = 28$, $y = -15$, and $z = -13$

It can be observed that,

$x + y + z = 28 + (-15) + (-13) = 28 - 28 = 0$

It is known that if $x + y + z = 0$, then

$$x^3 + y^3 + z^3 = 3xyz$$

$$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$
$$= 16380$$

Question 15:

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

$\boxed{\text{Area: } 25a^2 - 35a + 12}$	$\boxed{\text{Area: } 35y^2 + 13y - 12}$
I	II

Answer:

Area = Length × Breadth

The expression given for the area of the rectangle has to be factorised. One of its factors will be its length and the other will be its breadth.

(i) $25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$

$$= 5a(5a-3) - 4(5a-3)$$
$$= (5a-3)(5a-4)$$

Therefore, possible length = 5*a* − 3

And, possible breadth = 5*a* − 4

(ii) $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$$= 7y(5y+4) - 3(5y+4)$$
$$= (5y+4)(7y-3)$$

Therefore, possible length = 5*y* + 4

And, possible breadth = 7*y* − 3

Question 16:

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

Volume: $3x^2-12x$	Volume: $12ky^2+8ky-20k$
I	II

Answer:

Volume of cuboid = Length × Breadth × Height

The expression given for the volume of the cuboid has to be factorised. One of its factors will be its length, one will be its breadth, and one will be its height.

(i) $3x^2-12x=3x(x-4)$

One of the possible solutions is as follows.

Length = 3, Breadth = x, Height = x − 4

(ii) $12ky^2+8ky-20k=4k(3y^2+2y-5)$

$$
\begin{aligned}
&=4k\left[3y^2+5y-3y-5\right]\\
&=4k\left[y(3y+5)-1(3y+5)\right]\\
&=4k(3y+5)(y-1)
\end{aligned}
$$

One of the possible solutions is as follows.

Length = $4k$, Breadth = $3y$ + 5, Height = y − 1

$$
\begin{array}{r|l}
 & x^2-3x+2 \\
\hline
x+1 & x^3-2x^2-x+2 \\
 & x^3+x^2 \\
 & -\quad - \\
\hline
 & -3x^2-x+2 \\
 & -3x^2-3x \\
 & +\quad + \\
\hline
 & 2x+2 \\
 & 2x+2 \\
 & -\quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder

∴ $x^3 - 2x^2 - x + 2 = (x + 1)(x^2 - 3x + 2) + 0$

$= (x + 1)[x^2 - 2x - x + 2]$

$= (x + 1)[x(x - 2) - 1(x - 2)]$

$= (x + 1)(x - 1)(x - 2)$

$= (x - 2)(x - 1)(x + 1)$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

All the factors of 5 have to be considered. These are ±1, ± 5.

By trial method,

$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5 = 0$

Therefore, $x + 1$ is a factor of this polynomial.

Let us find the quotient on dividing $x^3 + 3x^2 - 9x - 5$ by $x + 1$.

By long division,

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
\hline
x+1 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -4x^2 - 9x - 5 \\
 & -4x^2 - 4x \\
 & + \quad + \\
\hline
 & -5x - 5 \\
 & -5x - 5 \\
 & + \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder

∴ $x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5) + 0$

$= (x + 1)(x^2 - 5x + x - 5)$

$= (x + 1)[(x(x - 5) + 1(x - 5)]$

$= (x + 1)(x - 5)(x + 1)$

$= (x - 5)(x + 1)(x + 1)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

All the factors of 20 have to be considered. Some of them are ±1,

± 2, ± 4, ± 5 ……

By trial method,

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33 = 0$

As $p(-1)$ is zero, therefore, $x + 1$ is a factor of this polynomial $p(x)$.

Let us find the quotient on dividing $x^3 + 13x^2 + 32x + 20$ by $(x + 1)$.

By long division,

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & 12x^2 + 32x \\
 & 12x^2 + 12x \\
 & - \quad - \\
\hline
 & 20x + 20 \\
 & 20x + 20 \\
 & - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder

$x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20) + 0$

$= (x + 1)(x^2 + 10x + 2x + 20)$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 10)(x + 2)$

$= (x + 1)(x + 2)(x + 10)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

By trial method,

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1 = 0$

Therefore, $y - 1$ is a factor of this polynomial.

Let us find the quotient on dividing $2y^3 + y^2 - 2y - 1$ by $y - 1$.

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y - 1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & 3y^2 - 2y - 1 \\
 & 3y^2 - 3y \\
 & - \quad + \\
\hline
 & y - 1 \\
 & y - 1 \\
\hline
 & 0 \\
\hline
\end{array}
$$

$p(y) = 2y^3 + y^2 - 2y - 1$

$= (y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)[2y(y + 1) + 1(y + 1)]$

$= (y - 1)(y + 1)(2y + 1)$

02 POLYNOMIAL

MIND MAP

Polynomials

Algebraic Identities

$(x+y)^2$	$x^2 + 2xy + y^2$
$(x-y)^2$	$x^2 - 2xy + y^2$
$x^2 - y^2$	$(x - y)(x + y)$
$(x+a)(x+b)$	$x^2 + (a+b)x + ab$
$(x + y + z)^2$	$x^2+y^2+z^2+2xy+2yz+2zx$ $= x^2+y^2+z^2+2(xy+yz+zx)$
$(x + y)^3$	$x^3 + y^3 + 3xy(x + y)$
$(x - y)^3$	$x^3 - y^3 - 3xy(x - y)$
$x^3+y^3+z^3-3xyz$	$(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$
If $x + y + z = 0$	(i) $x^3+y^3+z^3 = 3xyz$ (ii) $\frac{x^2}{yz} + \frac{y^2}{xz} + \frac{z^2}{xy} = 3$
$x^3 + y^3$	$(x+y)(x^2-xy+y^2)$
$x^3 - y^3$	$(x-y)(x^2+xy+y^2)$

Types

Terms

Polynomial	Example
Constant (or independent)	$4, -7/5$
Zero	Degree not defined (constant polynomial 0)
Monomial	$4x$
Binomial	$2x + 3$
Trinomial	$3x^2 + 7x + 2$

Degree

Polynomial	Example	Degree
Linear	$3x + 2$	1
Quadratic	$2x^2 + 3x + 1$	2
Cubic	$7y^3 + 6y^2 + 2y + 2$	3

Theorems

Factor Theorem

if p(x) polynomial of degree n>1, is divided by x−a, p(a) is the remainder

Divident = (Divisor × Quotient) + Remainder

Remainder Theorem

if p(x) is a polynomial of degree n>1, a: any real number

(i) if (x−a) is a factor of p(x), then p(a) = 0

(ii) if p(a) = 0, then (x−a) is a factor of p(x).

Zeroes of polynomial

Number that satisfies the equation

Example
P(x) = 2x + 1 find zeroes of the polynomial
P(x) = 0 2x + 1 = 0 x = −1/2

−1/2 is the zeros of the polynomial

Polynomial

Polynomial in one variable: $ax^3 - bx^2 - cx + d$

An algebraic expression of the form: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x^1 + a_0 x^0$

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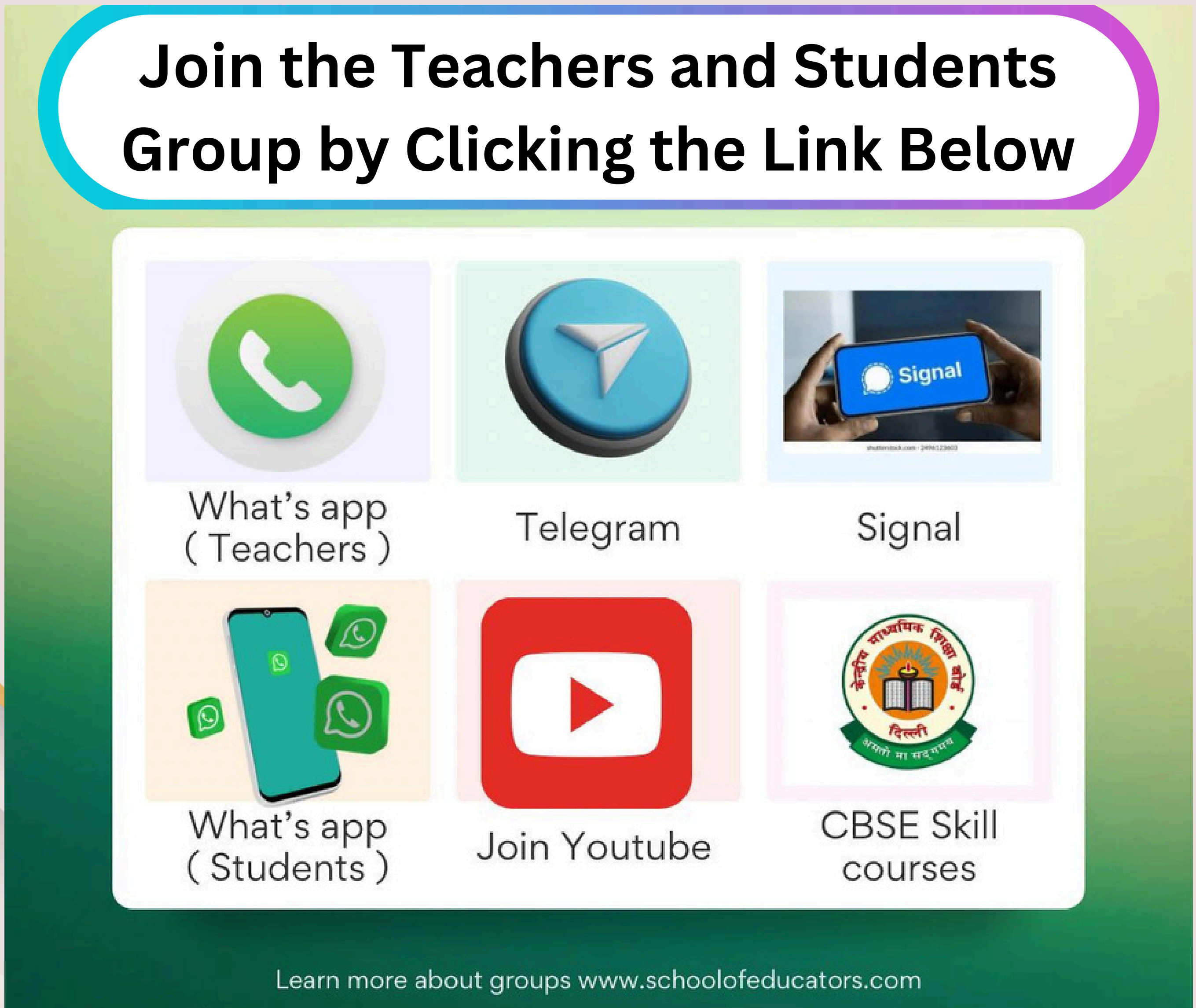

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Groups Rules & Regulations:

To maximize the benefits of these WhatsApp groups, follow these guidelines:

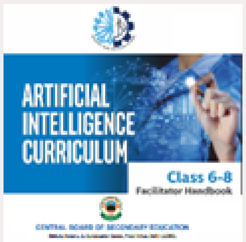
1. Share your valuable resources with the group.
2. Help your fellow educators by answering their queries.
3. Watch and engage with shared videos in the group.
4. Distribute WhatsApp group resources among your students.
5. Encourage your colleagues to join these groups.

Additional notes:

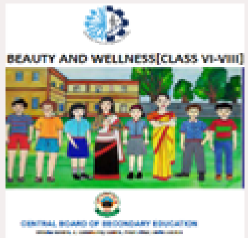
1. Avoid posting messages between 9 PM and 7 AM.
2. After sharing resources with students, consider deleting outdated data if necessary.
3. It's a NO Nuisance groups, single nuisance and you will be removed.
   - No introductions.
   - No greetings or wish messages.
   - No personal chats or messages.
   - No spam. Or voice calls
   - Share and seek learning resources only.

Please only share and request learning resources. For assistance, contact the helpline via WhatsApp: +91-95208-77777.

SKILL MODULES BEING OFFERED IN MIDDLE SCHOOL

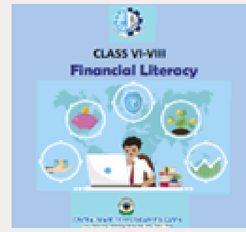


Artificial Intelligence



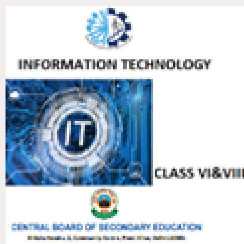
Beauty & Wellness

Design Thinking & Innovation



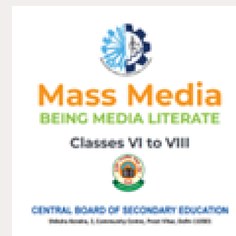
Financial Literacy

Handicrafts

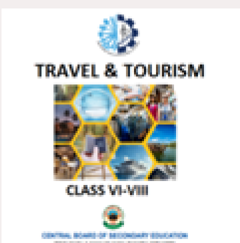


Information Technology

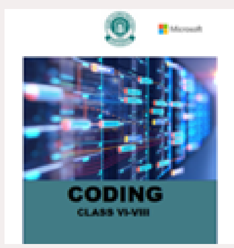
Marketing/Commercial Application



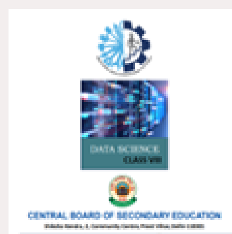
Mass Media - Being Media Literate



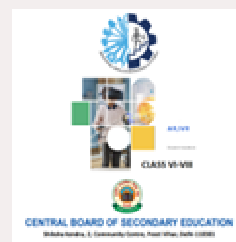
Travel & Tourism



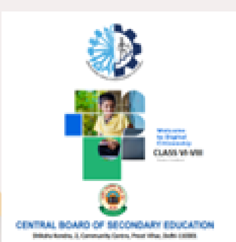
Coding



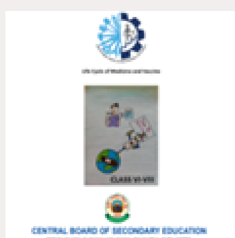
Data Science (Class VIII only)



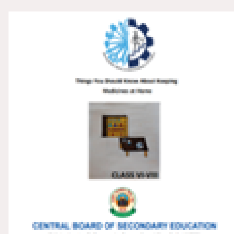
Augmented Reality / Virtual Reality



Digital Citizenship

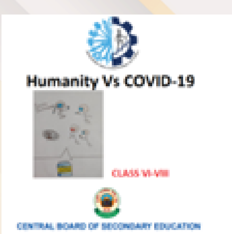


Life Cycle of Medicine & Vaccine

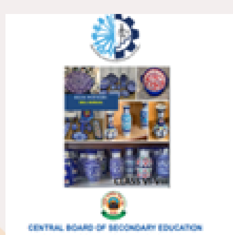


Things you should know about keeping Medicines at home

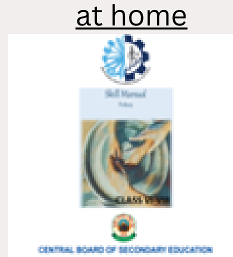
What to do when Doctor is not around



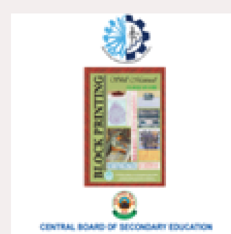
Humanity & Covid-19



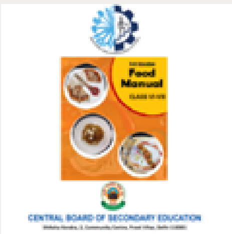
Blue Pottery



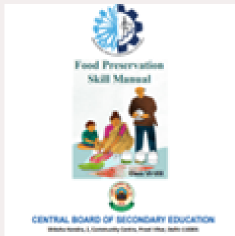
Pottery



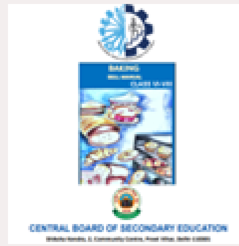
Block Printing



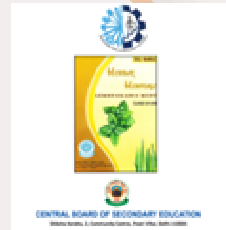
[Food](#)



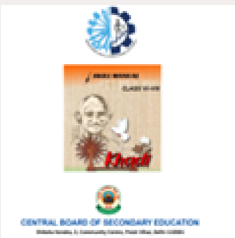
[Food Preservation](#)



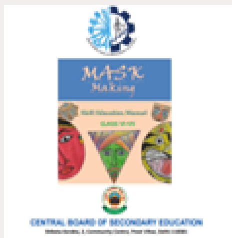
[Baking](#)



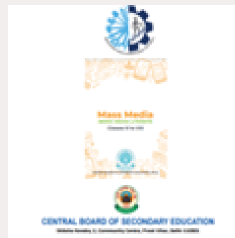
[Herbal Heritage](#)



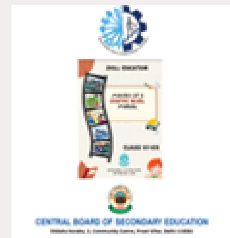
[Khadi](#)



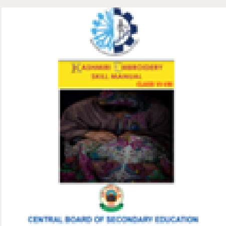
[Mask Making](#)



[Mass Media](#)

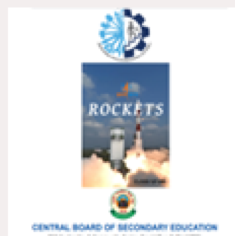


[Making of a Graphic Novel](#)

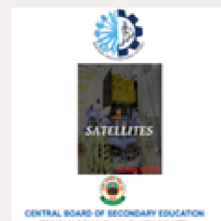


[Kashmiri Embroidery](#)

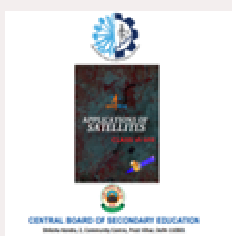
[Embroidery](#)



[Rockets](#)



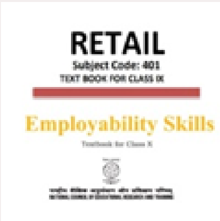
[Satellites](#)



[Application of Satellites](#)

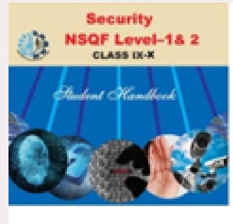
[Photography](#)

SKILL SUBJECTS AT SECONDARY LEVEL (CLASSES IX – X)

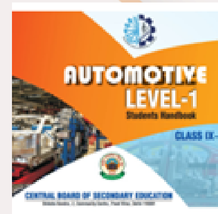


Retail

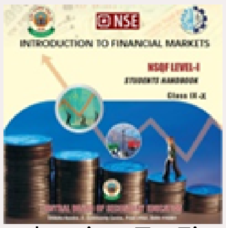
Information Technology



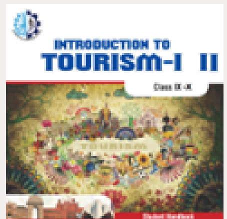
Security



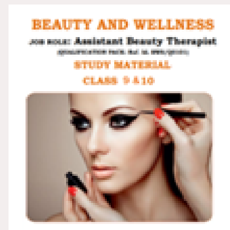
Automotive



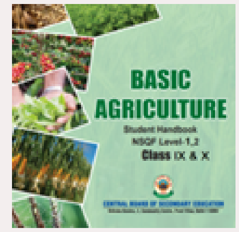
Introduction To Financial Markets



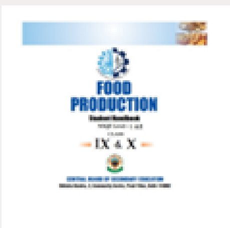
Introduction To Tourism



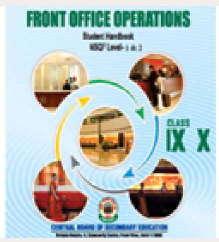
Beauty & Wellness



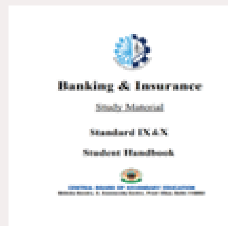
Agriculture



Food Production

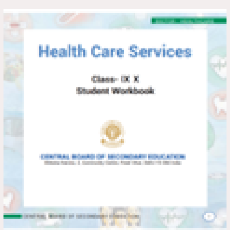


Front Office Operations



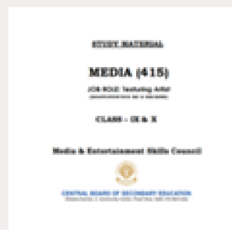
Banking & Insurance

Marketing & Sales



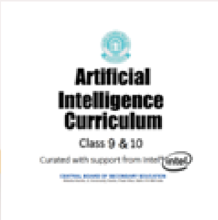
Health Care

Apparel

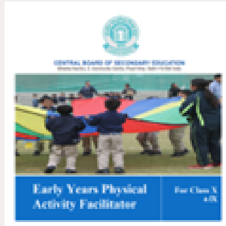


Multi Media

Multi Skill Foundation Course



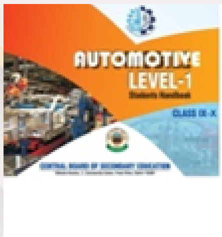
Artificial Intelligence



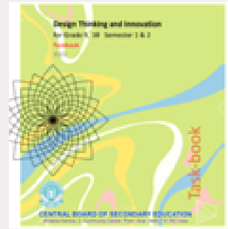
Physical Activity Trainer

Data Science

Electronics & Hardware (NEW)

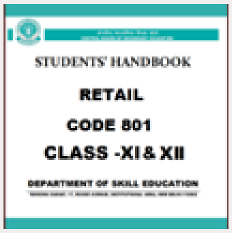


Foundation Skills For Sciences (Pharmaceutical & Biotechnology)(NEW)

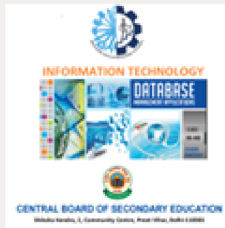


Design Thinking & Innovation (NEW)

SKILL SUBJECTS AT SR. SEC. LEVEL (CLASSES XI – XII)



Retail

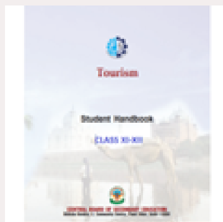


InformationTechnology

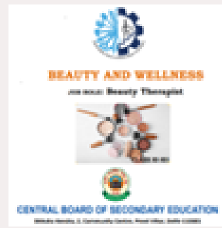
Web Application

Automotive

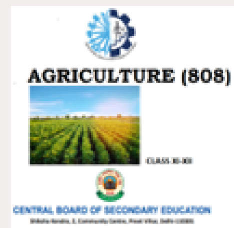
Financial Markets Management



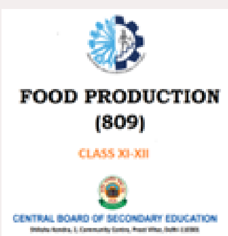
Tourism



Beauty & Wellness

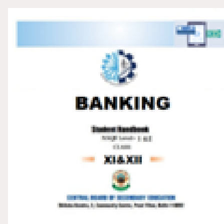


Agriculture

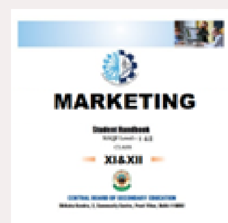


Food Production

Front Office Operations

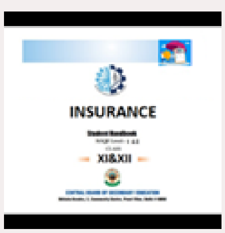


Banking

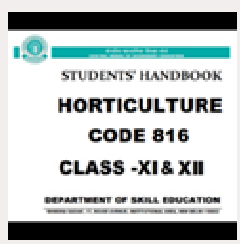


Marketing

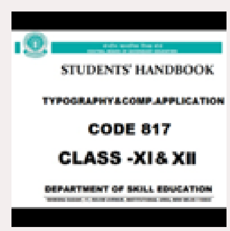
Health Care



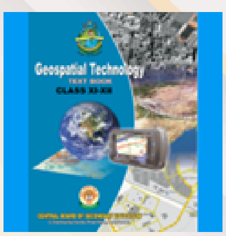
Insurance



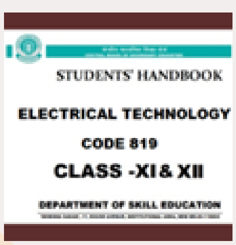
Horticulture



Typography & Comp. Application

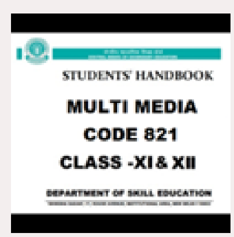


Geospatial Technology

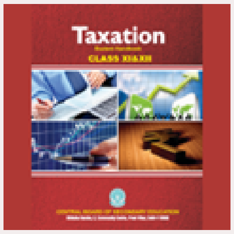


Electrical Technology

Electronic Technology

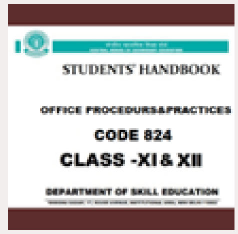


Multi-Media

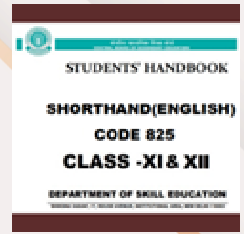


Taxation

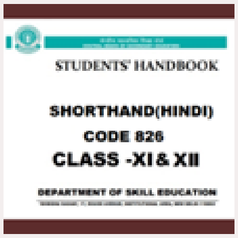
Cost Accounting



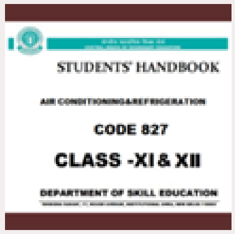
Office Procedures & Practices



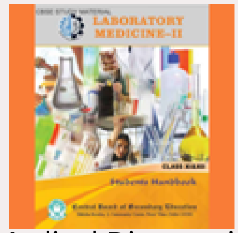
Shorthand (English)



Shorthand (Hindi)

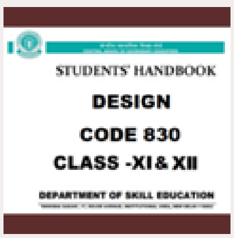


Air-Conditioning & Refrigeration

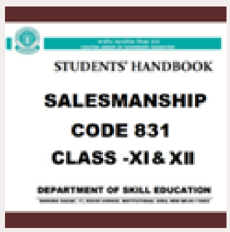


Medical Diagnostics

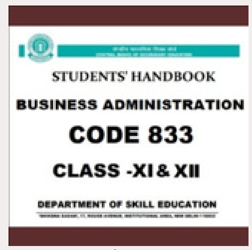
Textile Design



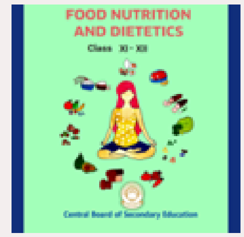
Design



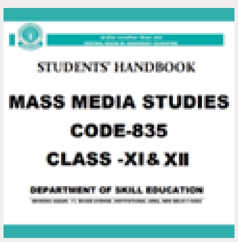
Salesmanship



Business Administration

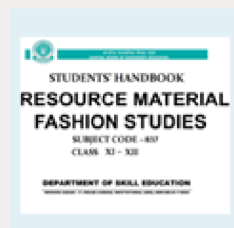


Food Nutrition & Dietetics

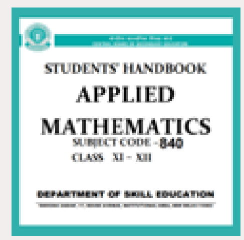


Mass Media Studies

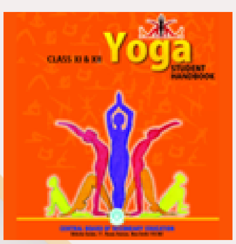
Library & Information Science



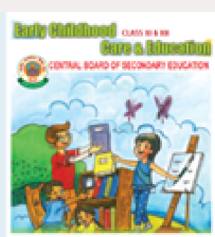
Fashion Studies



Applied Mathematics



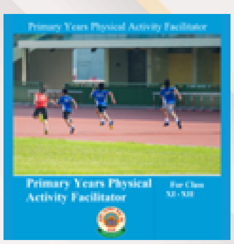
Yoga



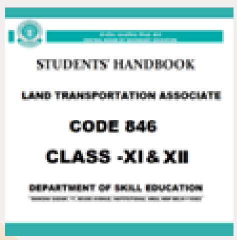
Early Childhood Care & Education

Artificial Intelligence

Data Science



Physical Activity Trainer(new)



Land Transportation Associate (NEW)

Electronics & Hardware (NEW)

Design Thinking & Innovation (NEW)

Subject Wise Secondary and Senior Secondary Groups IX & X

Secondary Groups (IX & X)

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Artifical intelligence

Senior Secondary Groups XI & XII

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Biology

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Class 8

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Class 9

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Class 10

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Class 11 (Sci)

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Class 11 (Com)

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Class 11 (Hum)

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Class 9

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Class 11 (Com)

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